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ABSTRACT

This study involved secondary teachers in the evaluation, modification, and use of a program built and experimentally tested in elementary schools. Program reactions were gathered from teachers who would eventually use the program, and evaluations were also obtained from instructors outside the experiment. Evaluations from both groups were similar. A three-group design was used in testing that consisted of partnership, treatment, and control teachers. No differences in degree of program implementation were found between partnership and treatment teachers, but both of these groups implemented more than the control instructors did. Among other findings, program teachers differed notably from control teachers in that program instructors used more problem-solving strategies. Further, significant differences in problem-solving scores were noted for program pupils when contrasted to students in control classes, and gains in student achievement were paralleled by higher problem-solving implementation scores in program instructional situations. This document covers general issues in program implementation and working with secondary teachers, and concludes with suggestions for future investigation. (MP)

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Experimental Research in Secondary Mathematics Classrooms: Working with Teachers

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Abstract

The research reported here involved secondary teachers in the evaluation, modification, and use of a mathematics program that had been built and experimentally tested in elementary schools. In addition to obtaining reactions to the program from teachers who would eventually use the program, evaluations were also obtained from secondary mathematics teachers who were not a part of the experimental program.

It was felt that teachers who were and were not asked to use the program might use different criteria for evaluating and modifying the program. However, it was found that both groups of teachers made similar recommendations. Still, the comparison between two groups of teachers proved valuable because it suggested context differences in school districts that appear relevant to implementation success.

In testing the program, a three-group design was utilized. The implementation of the program was compared for partnership teachers (teachers who had a chance to modify the program); treatment teachers (teachers who were asked to use the modified program, but who played no role in program development); and control teachers. No differences were found in the degree of program implementation for partnership and treatment teachers. However, both groups of teachers implemented more aspects of the program than did control teachers.

It was found that program teachers differed notably from control teachers in that they used more problem-solving strategies than did control teachers. These implementation differences are reflected in student performance. That is, weak differences were found in general achievement differences in favor of students in program classrooms (general program implementation did not differ sharply in program and control classrooms). However, significant differences in problem-solving scores were evidenced for program students in contrast to students in control classes. These gains in student achievement were paralleled by higher problem-solving implementation scores in program than control classrooms.

General issues in program implementation and working with secondary teachers are discussed in the report. Finally, detailed suggestions for future program modification and research are discussed.

Acknowledgements

We want to thank the National Institute of Education for the basic grant support which allowed us to conduct this project and for the freedom and flexibility in adapting the research plan to fit our evolving definition of the problem. Although we acknowledge the fundamental support of our work by NIE, the ideas presented here are our own and do not necessarily reflect the opinion or the endorsement of the National Institute of Education.

We also want to acknowledge the assistance of the Graduate School and the Center for Research in Social Behavior for general logistical support. The work of Patricia Shanks in typing (and retyping) the final report was done expertly and we are very appreciative of her efforts. Janice Sato and Terry Brown also typed parts of the manuscript as well as helping us to prepare the manuscript in numerous other ways. Gail Hinkel was very helpful in reading the manuscript and in making editorial suggestions.

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Finally, we want to thank two individuals who played valuable consulting roles in the research. Dr. Dee Ann Spencer-Hall was very helpful in obtaining teacher reactions to the program and in helping us to examine the potential contextual effects of using different implementation locations. Dr. Jere Confrey played a very valuable role by evaluating the extent to which mathematics presented in the textbook matched the mathematics assessed in the standardized achievement test and by helping to interview project teachers about their views of mathematics.

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Previous Research

Naturalistic Study

The research presented in this report is based upon three previous studies which were supported by two grants from the National Institute of Education. In 1975 we completed a large observational study of teaching effectiveness in third-grade mathematics classrooms (Good and Grouws, 1975). The purpose of that research was to determine whether it was possible to identify teachers who were consistent (across different groups of students) and relatively effective or ineffective, using student performance on the Iowa Test of Basic Skills as an operational criterion. Furthermore, it was our intention to observe teachers who differed in effectiveness and to see if differences in their classroom behavior could be identified.

To identify patterns of teacher behavior which affected student learning, it was considered desirable to focus all initial observation upon classroom activity during the teaching of a particular subject. Mathematics instruction was chosen because of its importance in the elementary school curriculum (reading and mathematics are commonly accepted as the major curriculum areas in elementary schools). We also thought that more teacher and school variance would be associated with students' mathematics performance than reading performance. This assumption has now received empirical support from Coleman's analyses of data from the International Educational study (1975).

The research was conducted in a school district just outside the core of a large urban city. The student population was primarily middle-class, but a number of students from low- and high-income homes attended target schools. Teachers in the district were using the same textbook series and the district had a stable population.

Over one hundred third- and fourth-grade teachers were initially studied. The data unit for the investigation was individual students' scores on the mathematics section of the Iowa Test of Basic Skills. Data for teachers were then compiled by computing the mean residual gain score (from the scores for students) over consecutive years.

We identified nine teachers who were relatively effective (in terms of the operational definition of effectiveness being utilized) and stable on total math residual scores across two consecutive years. We also found nine teachers who were relatively ineffective and stable across two consecutive years. During the year of observation, the more and less effective teachers again maintained their relative patterns of achievement. Hence, these teachers were stable across three years.

Observational data were collected in forty-one classrooms, to protect the identity of the relatively effective and ineffective teachers. Approximately equal numbers of observations were made in all classrooms (6-7). Data were collected by two trained observers (both certified teachers) who worked full-time and lived in the target city. Each coder visited all forty-one teachers and made about one-half of the observations obtained in a given classroom. Furthermore, all observations were made without knowledge of the teachers' levels of effectiveness.

Four basic sets of information were collected in the study. First, time measures were taken to describe how mathematics instructional time was utilized. A second set of codings were low-inference descriptions of teacher-student interaction patterns. These data were collected with the Brophy-Good Dyadic System (1970). A third set of data were high-inference variables drawn from the work of Emmer (1973) and Kounin (1970). Checklists were also used to describe materials and homework assignments (Good and Grouws, 1975).

Detailed accounts of the procedures and results can be found elsewhere (Good and Grouws, 1975; Good and Grouws, 1977). In brief, teachers' ability to obtain relatively high residual mean scores appeared to be strongly associated with the following factors: (1) whole-class instruction; (2) generally clear instruction and availability of information to students as needed (process feedback, in particular); (3) a non-evaluative and relaxed learning environment which was task-focused; (4) higher achievement expectations (more homework, faster pace); and (5) classrooms which were relatively free of major behavioral disorders.

Teachers who obtained high student achievement test scores were active teachers. They presented students with a meaningful and clear presentation of what was to be learned, provided developmental feedback when it was needed, structured a common seatwork assignment, and responded to individual students' needs for help.

We were aware that our initial results were only correlational data and that they did not necessarily imply that differences between high- and low-achieving teachers caused student achievement. It could very well be that behaviors not studied in our observational research are more directly related to achievement (for example, more effective teachers plan more thoroughly and because of this, they are more task-focused, assign more homework), or that these teachers taught more actively because they had more energy or because of other personality characteristics. We felt that it was important to determine whether a more direct association could be established between the behaviors which were identified in our observational, naturalistic study and student achievement.

Experimental Study I: Teacher Training Program

We were pleased that some consistent differences could be found in correlational research between relatively effective and ineffective mathematics teachers. However, at that point we only had a description of how more and less effective teachers (in our sample) behaved differently. We did not know if teachers who did not teach the way more effective teachers did could change their behavior or whether students would benefit if teachers were trained to use new methods. With the assistance of another grant from the National Institute of Education, we began a new type of inquiry, to determine whether teachers could be taught the behaviors associated with higher pupil achievement, and whether such teacher training would improve the mathematics performance of students. (For detailed accounts of the procedures and results of this experimental work, the interested reader should consult our 1979 final report; Good and Grouws, 1979a.)

The training program. In writing the teacher training materials, our earlier naturalistic findings were integrated with the recent naturalistic research of others and with existing experimental research in mathematics education, and translated into an instructional program. Some of the variables we tested in our experimental program did not come directly from teaching behaviors measured in our earlier studies, but were instead based upon what observers had seen in classrooms. Still other variables (e.g., mental computations) came from experimental studies. The training program resulted in a 45-page manual for teachers. The program, as pointed out elsewhere (Good and Grouws, 1979a), is a system of instruction: (1) instructional activity is initiated and reviewed in the context of meaning; (2) students are prepared for each lesson stage to enhance involvement and to minimize errors; (3) the principles of distributed and successful practice are built into the program; (4) active teaching is demanded, especially in the developmental portion of the lesson (when the teacher explains a concept being studied, its importance, etc.). An overview of the program is presented in Table 1 (the entire program can be found in Appendix 1).

In the 1975 naturalistic study, emphasis was placed upon internal validity. We chose a relatively stable school district (few changes in teacher personnel) in which a common textbook was used in all classrooms, and where student populations were comparable across schools and classrooms. The reason for these controls was to exclude as many rival hypotheses as possible to the conclusion that teachers and teaching were affecting student learning. In the 1978 experimental study, external validity was emphasized. A more heterogeneous school population was sampled because we felt this would be a more legitimate test of the training program. Details of the training procedures, a description of the sample, and more details on the results of the experiment can be found in Good and Grouws (1979a, 1979b). A brief summary of the results follows.

Observers' records indicated that the experimental teachers implemented the program very well (with the exception of certain recommendations concerning how to conduct the development portion of the lesson). Because experimental teachers did use the program and because the frequencies of their behaviors related to program recommendations varied significantly from those of control teachers, it was possible to determine how the experimental training and subsequent teaching activity influenced student achievement and attitudes. For implementation information see Table 2.

Pre- and post-testing with the SRA standardized achievement test indicated that after two and one-half months of the program, students in experimental classrooms scored five months higher than those in the control classrooms. Results are presented in Table 3. Regular end-of-year testing by the Tulsa public school system indicated that approximately three months after the program had ended, the experimental students were still performing better than the control students. We also utilized a content test (constructed by Dr. Robert Reys) which attempted to more closely match the material that teachers were presenting than did the standardized test. The results on this test also showed an advantage for experimental classes, although differences between control and experimental classrooms were not as large as they were on the standardized achievement test.

Results of pre- and post-testing on a ten-item attitude scale revealed that experimental students reported significantly more favorable attitudes at the end of the experiment than did control students. Also, it is important to note that anonymous feedback from teachers in the project indicated that they felt the program was practical and that they planned to continue using it in the future.

Research elsewhere indicate that teachers have a favorable reaction to the program, even when it is presented and discussed without the involvement of the developers (Keziah, 1980; Andros and Freeman, 1981). Obviously, if teachers are to continue using any program, they must feel comfortable with it; there appear to be sufficient data to suggest that teachers are reasonably pleased with the training program we have developed.

Interactions of student and teacher characteristics with the treatment program. To explore achievement patterns more fully in terms of student and teacher characteristics, it was considered important to define teacher and student types more broadly. Much of the responsibility for the conceptualization and analysis in this project was assumed by Dr. Howard Ebmeier, and more detail can be found in his dissertation (Ebmeier, 1978) and in a journal article about these results (Ebmeier and Good, 1979).

To develop student typologies, an instrument (Aptitude Inventory) was designed to assess student characteristics which might interact with key features of the treatment program, identifiable teacher characteristics, and/or classroom procedures. To obtain teachers' views of the characteristics, organization, and typical activities of their classrooms, a questionnaire was developed (Teaching Style Inventory). The Aptitude Inventory was administered to all students in the sample and the Teaching Style Inventory was administered to each teacher.

Cluster analysis was used to group students and teachers each into four types. The statistical properties of the 4 x 4 x 2 factorial design were tested using analysis of variance procedures. The residual scores on the SRA mathematics tests were the dependent variable. All main and interactive effects among and between teacher types, student types, and treatment types (control or experimental) were statistically significant. Details on the results and their interpretation can be found elsewhere (Ebmeier, 1978; Ebmeier and Good, 1979). Only a few comments on the findings will be presented here.

The results suggested that the treatment generally worked (i.e., the means in each cell were in favor of the treatment group), but the program was clearly more beneficial for certain combinations of teacher and student groups than for others. The data collectively indicated that teachers who implemented the model got good results, yet some teacher types chose to use more facets of the program than did other teachers.

One of the most interesting findings of the study was the interactions between teacher type and treatment type. There was a strong teacher effect in the treatment condition that was not found in the control sample. This interaction occurred for types 2 (experienced/unsure) and 3 (educated/secure) teachers, but not for the other two teacher types.

Since people are more likely to adopt and internalize ideas which are consonant with their existing beliefs, one could predict that teachers who already believe in an active instructional model or teachers who are unsure using their present instructional strategies would be most likely to implement the experimental treatment program if requested to do so. These results are consistent with the findings here. Although the results of the experimental field study have strong implications, they must be interpreted in light of the evidence that the experimental treatment worked better for some combinations of teachers and students than it did for other combinations.

As we noted in our 1979 final report, at that time Mr. Terrill Beckerman was completing a dissertation examining other interactions of the treatment program with student characteristics. In particular, his dissertation was an attempt to form student clusters on the basis of teacher descriptions of students rather than based upon student descriptions, which were used in the Ebmeier study. A detailed account of Dr. Beckerman's work can be found elsewhere (Beckerman, 1981); however, a brief description of his work is included in Appendix 2.

This material has been appended because it is not directly relevant to the work being presented in this final report. However, these results are interesting and broaden the findings presented in our 1979 final report. Another important addition to the 1979 report is found in Appendix 3. In writing the 1979 report, we indicated our intentions to examine the effects of the treatment upon students at different achievement levels. The interested reader can find these results for our fourth-grade experimental study in Appendix 3.

Experimental Study II: Verbal Problem Solving

Much of the research described briefly above was still in progress when the decision about the second experimental study had to be made (e.g., resources had to be allocated well in advance of data collection, the school district needed to be informed about the nature of the second field experiment, etc.). In retrospect, we feel that our ultimate decision to shift our concern to the development of a second treatment program (verbal problem solving) and to test older students (sixth, as compared to the fourth graders in the first field experiment) in the same school district was adequate, but perhaps not optimal.

We debated several possible topics for study. We considered an affective treatment; however, the fact that achievement gains appeared not to be coming at the expense of students' affective reactions suggested that there were no compelling reasons to proceed in this direction. A second issue related to the field experiment data was the relatively poor implementation of the development phase of the lesson. However, modifying this aspect of the program seemed too time-consuming. We also considered refining the treatment to make it more suitable for certain types of students and teachers. At this point, however, the Ebmeier typology work was still in an early stage of data analysis and although his initial work indicated that important interactions were occurring, it seemed premature to assess their importance or meaning. Because of Ebmeier's initial interesting results, we did make the decision to devote extra resources so that he and Beckerman could pursue their analyses of existing typologies more fully.

Ultimately, we decided to study verbal problem solving. As mentioned previously, two dependent measures of achievement were utilized in the first experimental study, the SRA achievement test and a special content test which Dr. Reys had designed. The reliability of Dr. Reys' instrument as a whole was acceptable, and the test showed that experimental students' achievement was superior to that of control students. However, the reliability for the three subtests of the instrument (knowledge, skill, and problem solving) indicated that only the skill subtest had adequate reliability for separate analysis (and on this subtest, the achievement of the treatment group surpassed that of the control group). In examining the means of the other two subtests, we found that treatment students did better controls on the knowledge items, but that there was little difference between the two groups on the verbal problem-solving test. However, it was impossible to tell whether the similar performance of the two groups was real or only a function of poor reliability (too few items). We were disappointed in these findings, because we felt that if mathematics knowledge is to be applied to "everyday life," students need practical problem-solving skills (e.g., the ability to determine whether the 12-ounce or 16-ounce package is the better buy). Unfortunately, extant literature on instructional behavior and students' performance on verbal problem solving did not provide any consistent orientation or procedure for classroom practice.

There was no data base for building a treatment program, especially data resulting from naturalistic studies of classroom teachers. Because we thought it was important to understand and to possibly improve students' ability for solving relatively simple verbal problems, we decided to make a systematic effort to develop testable instructional strategies in this area.

We therefore decided to shift our concern and to broaden the instructional program by adding a section on verbal problem solving. The first task was to develop a training manual detailing instructional strategies which teachers might use to teach students verbal problem-solving skills. The five techniques which teachers were requested to use were problems without numbers, writing verbal problems, estimating the answer, reading verbal problems, and writing open sentence problems. Discussion of these strategies and related research can be found elsewhere (e.g., Suydam and Weaver, 1970). A complete copy of the training manual is in Appendix 4.

It was also necessary to make three related decisions: (1) whether to test the instructional materials associated with verbal problem solving ith or without the program that had been designed for the first field experiment; (2) at what grade levels to test the program; (3) whether to observe or not.

It seemed important to determine whether gains in other mathematics knowledge and skill areas, such as those in Experiment I, could be obtained while students' performance on verbal problem-solving skills was also improved. Hence, it seemed more reasonable to expand and test a comprehensive program rather than to test only a verbal problem-solving strategy. In retrospect, we feel reasonably good about this decision, since subsequent research has suggested that the verbal problem-solving training manual does not appear to have effects independent of the broader treatment program (Engelhardt, 1980).

The grade level decision was a relatively straightforward one. We could have tested the program at the fifth-grade level and thereby have gained the advantage of studying the same students over consecutive years. However, the movement from school to school within the student population was relatively high. This meant that teachers would have some fifth-grade students who had been in the program and some who had not. To avoid this confusion, we decided to test the modified program at the sixth-grade level. We assumed that it would be possible to test the program on an "uncontaminated" population of classrooms, even though we stayed in the same school district. Using a sixth-grade sample also provided an older population upon which to test various questions about the general program.

The final decision we had to make concerned the role of observation in the field experiment. Limited funds, the fact that new observers had to be trained, and our interest in devoting resources to building a new treatment program (as well as exploring the existing typology data), collectively influenced our decision about whether to have limited observation or none at all. We ultimately decided to test the expanded program without classroom observation, because the generalizability of the findings would be increased if positive results were obtained. Our interest in expanding the results (not making observation a necessary part of the treatment) was supported by the results of Anderson, Evertson, and Brophy (1979), which suggested that their treatment had an effect upon student achievement which was not moderated by the presence of observers.

Unfortunately, despite our efforts to secure an "uncontaminated" population of sixth-graders, a degree of potential contamination was present in the design. In part, we were "victimized" by our previous success. The school district was sufficiently impressed with the results of the first study that they wanted all fourth-grade teachers to be exposed to the model. Due to this dissemination, as well as our own debriefing of control teachers in the first experimental study, program descriptions of the first experimental treatment were present in both treatment and control schools and hence, potentially available to sixth-grade teachers.

Procedures: Field Experiment II

The second field experiment was conducted one year after the first field experiment and in the same school district. The expanded program (the training manual used in experiment one plus the verbal problem-solving manual) was evaluated in thirty-six sixth-grade classrooms, and the general design and training procedures were the same as those utilized in the first field experiment (see Good and Grouws, 1979a, for complete details).

The only exception to the similarity of conditions between field experiment one and two was a major one. In the first field experiment, all teachers were using a semi-departmentalized structure (teachers taught only two or three different subjects a day). In the second field experiment, three organizational patterns were represented in the teacher sample. Some teachers utilized the semi-departmentalized structure; other teachers taught only math as a special subject (sixth-grade teachers taught math to several different sixth-grade classes); and some teachers were in open classes (where team teaching and individualized instruction were prevalent).

The semi-departmentalized structure and math-as-a-special-subject organizational patterns seemed to be consistent with the basic data base from which the project had been developed. The open classroom structure was not. However, the school district expressed interest in including some open classrooms in the design in order to have teachers exposed to the rationale for the active teaching aspects of our program. We included these teachers in the design, but emphasized both to administrators and to teachers during the training program that the treatment would be conceptual rather than operational (if teachers became interested in certain aspects of the program, the extent and form of adoption would be left to them).

Results of Experimental Field Study II

The raw means and standard deviations for the SRA (pre- and post-test) and the problem-solving post-test, by treatment condition and by organizational structure, are presented in Table 4. As can be seen, student performance increased from pre to post in all cases on the forty-item SRA test. Furthermore, all treatment groups surpassed the performance of the equivalent control groups. Two of the three treatment groups had higher mean performances than equivalent control groups on the problem-solving test. It should be noted that the exception, the open treatment classes, had the lowest pre-test score on the SRA.

Also, as can be seen in Table 4, the mean pre-test SRA scores for control teachers were generally lower than the scores in the equivalent treatment group. An exception occurred in the math-as-a-special-subject classes, where the pre-SRA mean scores of the treatment classes slightly exceeded those of control classrooms. To reiterate, in terms of raw gains, the treatment groups' performance was generally superior to that of the equivalent control groups.

The results of the formal analyses using adjusted mean scores indicated that the performance of the treatment group was not significantly higher than that of the control group on the post-SRA test, using the pre-SRA test as a covariant (with all forms of classroom organization included in the analysis). Similar results were also obtained when open classrooms were excluded from the analysis.

A similar analysis was performed on the problem-solving test (using the pre-SRA test as a covariant) to compare the significance of adjusted means across all treatment and control classrooms. This analysis indicated that the performance of the treatment group exceeded that of the control group in a way that approached significance (p = .10).

Earlier it was mentioned that we had some reservations about including open classroom teachers in the study because the program had not been designed for such settings. When open-space teachers were excluded, the comparison on the problem-solving test revealed that the treatment group's performance was significantly superior to that of the control group (p = .015). These results are presented in Table 5.

Other student and teacher data. Student affect was measured by the same ten-item instrument which was used in the first field experiment. The data suggested that the affective reaction was similar for both groups and that the treatment had no meaningful impact on student attitudes.

Reactions of the treatment teachers were assessed confidentially two months after the program had ended. The overall affective reaction of experimental teachers (sixteen of the teachers responded) was extremely positive. Questionnaire responses revealed that two-thirds of the participants continued using all aspects of the program at or very near the initial level recommended by the project directors. After the program ended, ten teachers were still including verbal problem solving in their curricula and thirteen were implementing the prescribed development phase at least three times a week. Fifteen teachers continued to assign homework a minimum of three nights a week, and thirteen were conducting weekly and monthly review sessions.

At the debriefing session we provided control teachers with a copy of the program manual. Two months later we assessed their reaction to these materials. We did this for two reasons. First, we wanted to see how teachers who had been exposed to the program but who did not use it would evaluate it. Were the favorable comments of experimental teachers due to the fact that they had used the program and hence felt obligated to recommend it? We also wanted to see how new the various aspects of the program were to control teachers. Their responses indicated that they were familiar with most of the recommended teaching techniques, and two or three of the control teachers said their supervisors had advocated that they use a directed lesson. Such responses suggest to us that at least in some cases, control teachers were using parts of the program.

Seventeen of the nineteen control teachers responded to the questionnaire. Five control teachers reported they had carefully read the general manual and the verbal problem-solving manual. Five others had read both manuals quickly and six had at least skimmed them quickly and had thought about the highlights. Responses revealed that there was considerable correspondence between the teaching methods control teachers were already using and those requested by the program. Eight teachers reported they were already utilizing the prescribed development and seatwork aspects of the program, and were also teaching their classes as a whole. At least five more teachers reported general overlap between the program and what they had been doing, for each category except the verbal problem solving. It seems reasonable to suggest that the general achievement of students was not enhanced by the experimental program partly because control teachers were using many aspects of the program, although we do not have observational data upon which to verify that point. Still, it is clear that the verbal problem-solving material was basically unique to control teachers (the new part of the present program and the part that was disseminated in the school system only to experimental teachers in this study). The experimental program did have positive effects upon students' verbal problem-solving skills.

More detailed discussion of the teachers' positive comments about the program is reported elsewhere (Good and Grouws, 1979a); however, it is important to note that teachers generally saw the program as valuable, whether they participated in it or not, and that these results have been replicated elsewhere (Keziah, 1980; Andros and Freeman, 1981). It may be instructive to list some of the negative comments that teachers made about the treatment program. When treatment teachers were asked the free-response question, "What were the weakest or most confusing parts of the program?", five teachers said that they had difficulty using it with classes in which there was a wide range in student ability. Some of these teachers felt the program was particularly difficult for low-ability pupils. Six teachers thought that there was not enough time allotted on a daily basis to complete all phases of the program. In response to this same question, only three control teachers listed weaknesses: it was hard for low achievers; there was not enough time to complete all parts of the program daily; and it was hard to get pupils to do homework on a daily basis. In general, sixth-grade teachers were supportive of the program (as had been fourth-grade teachers in the previous experiment) but they were able to report some perceived weaknesses.

The Present Study: Experimental Work in Junior High Classes

Considering results of the two earlier experimental studies, we were very much interested in expanding our inquiry to secondary classrooms. Unfortunately, at the time we were writing the proposal, there was very little process data which described the normative aspects of mathematics teaching in secondary settings. There were numerous conceptualizations about adolescent culture and student development, and general information about secondary schools (e.g., Coleman, 1961; Campbell and McSweeney, 1970; Metz, 1978).

Fortunately, what data did exist were largely consistent with our treatment program. For example, McConnell (1977) reported that the following teacher behaviors correlated with student learning in high school algebra classes: task orientation, clarity, enthusiasm, and frequent teacher talk. These variables were very similar to the teaching behaviors we were testing in elementary schools. Furthermore, our emphasis on the development portion of the lesson also had some empirical support on the secondary level (see, for example, Zahn, 1966).

Our elementary school data were also largely consistent with perhaps the most comprehensive source of information related to effective mathematics teaching in junior high schools (Evertson, Anderson, Anderson, and Brophy, 1980). In this study, intensive observational records of twenty-nine mathematics teachers in fifty-eight classes were correlated with student achievement data. The findings from this research overlap considerably with most aspects of our existing treatment program. These researchers found that effective instruction in junior high math classes was characterized by high academic orientation, relatively more whole-group instruction, frequent public recitation and discussion, active student involvement, and maintenance of a rapid pace.

Their results are also consistent with our program and findings in other ways. Both sets of data agree that successful mathematics teachers are more active in both public (development/discussion, recitation) and private (seatwork) parts of the lesson. Furthermore, both research programs illustrate that appropriate uses of monitoring and accountability are associated with student achievement, and both suggest that the relationship between lesson parts is critical (e.g., the amount of seatwork time is less important than how well students are prepared for it).

There are a few minor differences between the two sets of data. For example, the use of praise appears to be somewhat more important at the junior high level than our research in elementary schools suggests.

Other work in secondary schools (e.g., Stallings, et al., 1978), although not collected in mathematics classrooms, also indicates that active and structured teaching is practical in secondary settings.

In the original grant proposal, we described a three-year project comprised of three distinct studies. We would first conduct a treatment study (simply asking mathematics teachers to implement the existing program) and would use the results and the responses of these teachers (after they had utilized the program) to build a modified and perhaps more sensitive mathematics program for use in secondary settings.

In the second study, with the assistance of a new sample of teachers, we had planned to actively involve teachers in the modification of the training program. That is, the new secondary teachers would have been provided with program material and all related findings, including the results of the first secondary study, and comments by previous teachers who had used the program in fourth-, sixth-, and eighth-grade classrooms. In the third year of the study, we had proposed a retention study in order to determine the extent to which the treatment (of the year before) influenced students in their subsequent learning in mathematics.

Unfortunately, the research program reported here had to be modified to be conducted over a period of eighteen months (as opposed to three years) and with a reduced budget. Several important decisions thus had to be made and made rather quickly. Some of these decisions are described, because they may help other researchers who will be working with practitioners to think through potential problems.

Because of the time limit, it was possible to conduct only one major treatment study. Since we were committed to the idea of involving teachers in reviewing and planning the research, the question became, Under what circumstances could we do this? We found out about reduced funding in the summer and had to address the issue of involving teachers at that time. We had two alternatives: we could work with the teachers in a very quick manner and be able to begin the treatment in the fall, or we could use the fall semester as a way to become acquainted and work with teachers and conduct the experiment in the spring semester.

We chose the former, less-involved, partnership arrangement with teachers in order to begin the experiment in the early fall. We thought that secondary teachers might be less responsive than elementary school teachers to research (as described in the popular literature), and that once routines (and plans) were established in the school year, both secondary teachers and students would be more hesitant about changing classroom procedures. Clearly, this assumption in itself is an empirical question and one which merits investigation.

In retrospect, we are not too disappointed with this decision. There are many ways in which a partnership model (working with teachers) can be tested and implemented, and the context of our study dictated that we use a minimum model, with relatively little time for joint decision-making and planning. Whether the results would be different under other conditions and with different samples again is a topic for future research.

After having made the decision to begin the program relatively early in the year, there were still many procedural options which were available to us. For example, we could have had three different meetings, reviewing one-third of the training program at each meeting. The teachers could try the program for two weeks and then come back for major consolidation meetings, with the option of revising large parts of the program. We could have paid for substitutes so that some of the partner teachers could observe one another and use this information for modifying the program. We want to emphasize here that external constraints were instrumental in only one decision we made concerning the design of the study. Other options we chose were largely our own and in retrospect, even the original constraint does not seem especially important because of the large number of researcher-teacher partnership arrangements which need to be tested.

After deciding to test a treatment program relatively early in the year, we chose to use a minimal partnership arrangement wherein the time for involvement between teachers and researchers was relatively limited, but the decision-making process was still open and all aspects of the program were subject to change. We were prepared to spend as much time with teachers as necessary in order to develop a program which all participants were willing to implement (but only as much time as necessary). We ultimately excluded from consideration models which would provide continuing feedback to teachers (the chance to observe or to be observed by fellow teachers) or the opportunity to modify the program in major ways once it had been initiated.

We were very interested in secondary teachers' opinions of the program and the types and extent of modifications they would suggest. In previous interview work with teachers, one of us had discovered that teachers often have interesting explanations for their classroom behavior, even though they appear to be unaware of certain aspects of their behavior (Brophy and Good, 1974). We also had been favorably impressed by the work that Bill Tikunoff, Betty Ward, Gary Griffin, and others (working at the Far West Laboratory) had been doing in building partnership relations with teachers, and had seen in draft form some of the interesting work that had been produced by teachers (Behnke, et al., 1981). Although we wanted to work with teachers to modify a program rather than provide resources for teachers to do their own research work, we were encouraged by the potential benefits of involving teachers in program change.

We were also specifically interested in learning how secondary teachers would react to the program, because the recommended instructional techniques had largely resulted from our observation of elementary school teachers. We wanted to learn from teachers whether certain aspects of the program might be inoperative in secondary classrooms.

Prior to meeting with the teachers, we talked with Drs. Carolyn Evertson (then at the University of Texas) and Perry Lanier (Michigan State University) so that we could include their insights in potential modifications of the program. Both researchers were conducting large-scale studies with secondary mathematics teachers and we wanted to take advantage of their research experience. We did not want to provide this information to teachers in advance of our meeting because it could bias their initial impressions and reactions to the program (we didn't want to overload them with experts' opinions). However, we did intend to use Evertson's and Lanier's recommendations at the end of our decision-making conference with teachers, if necessary ("Here is what some other people have said about the program...what do you think of the potential value of their comments?"). As it turned out, both researchers thought the program would be perceived by junior high teachers as similar to what they were already doing and most of their suggestions were related to modifying the program to fit a more mature and responsible secondary-school student population. Several of their comments about modifying the program (e.g., the need for systematizing evaluation standards) were not utilized because of wide variation in teacher opinion. Most of the suggestions which the researchers made were also made by classroom teachers during our meetings with them (including the need for common evaluative practices).

The Planning Meeting With Teachers

Six teachers were randomly selected to be partnership teachers from the volunteer sample of teachers who were willing to participate in the project (the sample will be described below). Prior to attending the meeting, teachers were given both the general treatment manual and the verbal problem-solving manual and were asked to read and critique both manuals. In addition, teachers had an evaluation sheet to fill out and bring to the meeting. We wanted to determine what each teacher thought about the program prior to discussion. From our knowledge of group discussion literature, it seemed probable that some attitudes of individual teachers might be affected by the particular teachers who happened to speak first or the intensity of the presentation of individual teachers. The raw response forms that teachers brought to the meeting are presented in Appendix 5. As can be seen from examining these protocols, the teachers were basically supportive of the program and were willing to attempt to implement it when the session started.

At the beginning of the meeting, each of the principal investigators made a brief five-minute presentation about the scope of the project. Some of the comments by one investigator follow: "At this point, what we are doing is turning to teachers, the experts in secondary education, because this program was developed for elementary school usage. We're very interested in your criticisms, problems that might develop when the program is used in secondary schools, and we're willing to adapt it as necessary. This meeting is an open invitation for you to react to the program and to accept what appears to be useful and to revise parts that need to be changed. If it seems essentially workable and testable in its present form, that too is okay. We really don't have an agenda other than to explore the methods and get your reactions about what it would take to make the program work in secondary schools. We are taping the proceedings...in order to have a public record of what we've talked about and also to be certain that we remember all of the comments and suggestions that are made."

Comments from the other investigator included the following. "We appreciate your being here and we have enjoyed previous work in the elementary schools. In general, this program in the past has had good results, both in terms of student achievement and attitudes as well as teacher reports about the program. Elementary school teachers seem to be very satisfied with the program. I think the gains the students were making influenced how the teachers felt about the program and whenever students show interest in mathematics and make some achievement gains, that makes us teachers feel good. We're pretty excited about moving into the eighth-grade level. I think we're all aware of the fact that there is quite a difference between elementary and junior high school settings. I guess that's the purpose of this meeting, as Tom was saying. We would really like your sincere thoughts about parts of the program that you think work well or your thoughts about parts that might be revised as well as your recommendations about how we can improve the program to make it practical for use with junior high students. As Tom said, we really consider you to be the experts. You teach eighth-grade kids on a daily basis. I've taught in junior high in the past, but I'm aware that students have changed over the past few years. Without further ado, we'd like to start getting your comments. First, however, we'd like for you to pass in your written comments."

In other remarks we attempted to point out to partner teachers that though the program had worked reasonably well in elementary schools, we had no data base in secondary schools. In essence, we wanted teachers to know that we were very interested in making any and all modifications necessary to make the program work well in secondary schools, and that we encouraged their participation and especially their criticisms of the program.

After we had made our remarks, one of the participating teachers immediately asked the mathematics supervisor to react to the program and to the meeting generally. Among other things, the supervisor said, "To be clear, you're working directly with the University of Missouri and the school district doesn't control any of the factors of the research. When I first read the program, my evaluation was that the components were important to what junior high teachers should be doing. Although teachers may not sequence all parts of the program like they are here, it seems similar to what many junior high teachers are already doing. I was kind of pleased that we went into the research, because it's hard to get involved in research where there are teacher variables and attempts to improve instruction. In general, I appreciate what they're trying to do and I think that the program is a very good one."

The mathematics supervisor was very positive (perhaps too positive) about the potential benefits of the program and the strong working relationship that we had shared with the school district in previous work at the elementary school level. We had requested a meeting with the teachers alone; however, the supervisor also came, and under the circumstances, it was awkward not to allow him to participate in the meeting. He had planned to be an observer only, and his responses then (and a few later in the meeting) were at the request of particular teachers.

What effect his presence had (if any) cannot be determined. However, in subsequent debriefing interviews at the end of the project, two of the partnership teachers were explicitly asked about the presence of the supervisor in independent sessions. These two teachers indicated that they viewed (and that they thought others felt the same way) the secondary mathematics coordinator more as a friendly consultant who was interested in helping and working with them. They did not view him as an administrator whose role was to control teachers. Still, his presence and his expression of positive affect may have reduced some criticisms that teachers wanted to make. However, such criticisms were not given on the sheets that teachers brought to the meeting (see Appendix 5) and there is thus no evidence to indicate that the supervisor had any effect upon the proceedings.

In some ways, his presence may have been useful. For example, when the meeting was turned over to teachers for their input, the very first question was, "How were we selected?". Another participant asked, "I know we're here, but how many schools are involved? Are all schools in the city involved?" The supervisor pointed out that all junior high schools in the city had an opportunity to participate in the project, except for some of the Title I schools, which were involved in another experimental program on math skills. The district did not want the two research programs interfering with one another. He then stated that the investigators had selected the six teachers randomly from a list of volunteer schools.

At this point, one of the participating teachers indicated that the supervisor's account was consistent with her experience. She further noted, "Well, the day that our principal talked to us, he made participation optional on our part. We all kind of sat there and looked at each other and finally I said, 'Do we need unanimous participation in our building?' and he said, 'Preferably.' You know, by that time, we were all saying, 'Why not?'". However, at least one teacher felt that he/she was coerced into participation (see Appendix 6).

Group Discussion

In general, the first few moments of the group discussion were spent talking about general procedural events (Why were we selected?) and the history of the project (e.g., stating again that no work had taken place in the junior high with our program). Following introductory remarks by the principal investigators and the mathematics coordinator, and general procedural discussions, individual teachers were asked to characterize the strengths and weaknesses of the program from their individual perspectives and to suggest changes that the group might want to consider in modifying the program. To minimize premature evaluations, we asked each teacher to present his or her reactions before we requested general comments and reactions to the ideas which were presented. (We did this in part because at an earlier meeting with secondary teachers, we found that some teachers dominated the discussion--more on this later.) However, it was difficult to adhere to this procedure and we were engaged in group conversation about the wisdom of certain strategies before all teachers had made their comments. Nevertheless, all teachers in the project did have a chance to present their critiques of the program before serious negotiation began about aspects of the program which would be changed.

After each teacher's presentation, group discussion followed, more or less moderated by one of the investigators. Essentially, this dialogue proceeded in the following manner, "Here's a suggestion for change; what do you think about it; what are the advantages and disadvantages?" The decision-making discussion seemed to perseverate on two general topics associated with the project, but not explicitly addressed in project materials. One topic involved the role of testing in the program; some teachers wanted testing to occur on a controlled schedule and other teachers felt that different topics and different types of classrooms necessitated different types and frequencies of testing. Some teachers were adamant that homework had to be graded, and other teachers felt strongly that they did not have sufficient time to grade homework assignments (i.e., they preferred a checking system rather than a grading system). In the end, no changes were made in testing or the grading of homework.

The teachers strongly felt that more time should be provided for review in the junior high setting. They thought that students needed to be actively involved in the review process rather than that review should be conducted as a totally teacher-dominated activity. Accordingly, we decided to extend the length of time allocated for the introductory phase to twelve minutes. Teachers also wanted to incorporate the verbal problem strategies into the program in a very systematic way. They emphasized that verbal problem-solving instruction should take place every day for ten minutes, and that just having students work a few problems at their desks was an insufficient strategy. One teacher pointed out, and others agreed, that teachers need to prepare in order to follow this program and that the verbal problems to be solved should be determined before each class period begins. Verbal problems selected must be carefully interfaced with the development lesson which follows. Another important change which resulted from the teacher meeting was that the time recommended for homework was doubled. In elementary schools the recommended homework assignment took fifteen minutes; here, the time increased to thirty minutes. We decided that assignments should be flexible enough to allow for varied rates at which individual students work. Teachers could allow some students to do a portion of their homework in the classroom, but all students would have to do at least some of their homework outside class. Another program change was that students could now be assigned a weekly quiz as part of their broad review time on Monday.

In general, the discussion produced many shared assumptions about what the program was and was not, even though some of the discussion did not lead to changes in the program. The modified aspects of the program and those parts of the program receiving most comment and discussion during the group meetings were summarized by one of the principal investigators and subsequently given to teachers for their approval and/or suggestions. The modifications of the program appear in the following two reports: (a) Teachers' Manual Addendum for Junior High Work; and (b) A New Procedural Summary for the Verbal Problem Solving Manual.

Teacher's Manual Addendum for Junior High Work

This addendum describes modifications for using the Missouri Mathematics Effectiveness Project Teacher Manual in junior high school classes. The modifications include substantive changes as well as minor adjustments. The changes resulted from a group meeting of junior high mathematics teachers who read the materials and then met to discuss the program. The following revisions reflect the collective thinking of the group.

During the introductory phase of the lesson, a number of things must take place: a brief review, checking homework, and some mental computation. There was agreement that teachers should move rapidly through this phase, because there is a tendency to spend too much time going over homework, and also because the review at this point is distinct from prerequisite skills in the development portion of the lesson. However, in light of the additional math time available at the junior high school level, it was decided to extend the time allotted for the introductory phase to twelve minutes, as shown on the appended Time and Summary Sheet.

The ten minutes following the introductory activities are designated for instruction in verbal problem solving, using strategies outlined in the Verbal Problem Manual. The group endorsed instruction in verbal problem solving, and several teachers pointed out that the ability to compute in isolation is of very little value unless students can use the skills in a variety of situations. It should be emphasized that the time devoted to verbal problem solving should involve the teacher teaching and the students participating in class discussion. This recommendation is not fulfilled by just having the students work a few word problems at their desks. It was pointed out by one teacher that teachers need to be prepared to teach in order to follow this program and that this means having the verbal problems to be used in this part of the lesson determined before the period begins. It was also mentioned that there is often a variety of textbooks available at school and these can be a very useful source of problems for this part of the lesson.

The importance of active, careful, and meaningful teaching of the topic for the day in the development portion of the lesson was affirmed. The necessity for a smooth transition from the development phase to the seatwork phase (where students work individually on a collection of problems or exercises at their desks) was mentioned, along with the comment that much time could be lost if this transition is not carefully managed. The program suggests that teachers control practice in the latter part of the development phase; that is, teachers should have students work a problem like the first problem in their assignment and then discuss and demonstrate its solution in front of the class. This procedure should be repeated several times until all the students seem to have the idea. This controlled practice helps a great deal in getting students started immediately after the seatwork assignment is given.

The procedure for seatwork as described in the Teacher's Manual was not changed. Teachers are to make sure that each student is working before providing individual help. During seatwork, the teacher should circulate about the room, supervising student work to assure that students are not practicing incorrect methods.

There was considerable discussion about how to hold students accountable at the end of the period. It was agreed that this could be done in several ways, including: (1) calling on some students to give their answers to particular problems; (2) checking the answers to the first few problems orally (and then occasionally taking grades); (3) calling on students and asking how many problems they had finished. Other methods are also possible. The important thing is that students do some problems in class while the skills and ideas are fresh in their minds and while help is available for those who need it.

There was general agreement that a homework assignment on Monday through Thursday was a reasonable requirement and that students this age could be expected to work on homework for longer periods of time than those specified in the manual. It was decided that assignments should take 15-30 minutes. This flexibility allows for the different rates at which individual students work and also permits teachers who like to give a combined seatwork-homework assignment to do so. That is, a given number of problems is assigned and students are allowed to work on them during the 10-20 minutes of seatwork in class; whatever problems are not finished are completed outside of class as homework. Under these circumstances, the assignment should be long enough to provide some homework for everyone.

Several teachers mentioned that junior high students need to assume more responsibility than they have previously, and that teachers should thus expect homework to be done on time. Some teachers suggested different ways of recording homework grades (e.g., on some days, scoring the papers and recording the grades, and on other days just checking off complete papers. In this way students would not know which procedure was to be used on a particular day). The way teachers handled collected homework was left to the discretion of individual teachers.

The discussion of the structured reviews every Monday centered on how they should be conducted and the flexibility in scheduling them. There was agreement that the review need not be solely lecture, but should include student interaction in the form of student questions, having students go to the board, or having students work a review problem at their desks with discussion following. The possibility of a weekly quiz during part of the review time was discussed and this was deemed acceptable. It was decided that teachers could have this flexibility in scheduling reviews, but when a review was not conducted on Monday, the day it was conducted should be recorded in the log.

The timing of quizzes and tests was discussed and this scheduling was left to the individual teacher, with the understanding that tests and quizzes would be noted in the daily log.

Finally, the topic of occasional variation from the time framework was discussed, and the consensus reached was that teachers should adhere as closely as possible to the guidelines in order to give the program a valid test, realizing that some variance from the guidelines may be unavoidable. However, in such cases, each teacher should make a concerted effort to adhere as closely as possible to the guidelines.

Procedural Summary
Verbal Problem Solving Manual

The ability to solve verbal problems, or word problems, is an important skill. In order to insure that verbal problem solving will receive systematic attention, each teacher is asked to spend ten minutes of every mathematics class period on this topic. Students are thus exposed to verbal problem solving daily, even when problem solving is not the principle objective of the lesson. This ten-minute period should be a time when the teacher teaches problem solving and not a time for students to sit passively in their seats and work a couple of verbal problems. That is, the teacher should actively model the solving process, have students work a problem, and then have a class discussion of ways to solve the problem, and so on.

Preparing to actively teach verbal problem solving is not always easy. To assist in the preparation of this part of each lesson, a Verbal Problem Solving Manual has been developed. It details five instructional strategies which seem to be associated with improved student performance in this area. We ask that you use one of these strategies each day.

The strategies in the manual can be used independently or in combination. Some will be more appropriate for particular problems or topics than others, and the choice of which strategy to use on a given day is left to the discretion of each teacher. Since each of the ideas has merit, and since there is value in using a variety of ideas, it is important to use each idea regularly. Whether one idea is used for a week at a time, or a different idea is used each day is not important. To insure that every idea gets some exposure, we ask that each idea be implemented at least once every two weeks.

The procedures outlined in the Verbal Problem Solving Manual are designed to be used with textbook problems and your textbook will be the primary resource for these problems. Other resources can include textbooks no longer in use, textbooks from lower grade levels, and problems based on information generated during class discussions, field trips, and so on.

A table summarizing the key points of each idea in the Manual is attached to aid you in viewing the program at a glance.

The Verbal Problem Solving Manual is the detailed resource to assist you in the application of the ideas.

Each teacher is asked to keep a record of the verbal problem-solving activities used. The log should include the amount of time spent on problem solving, the strategies used, the in-class and homework assignments on problem solving, and any exceptions or conflicts which affect the program. By exceptions we mean situations which arise from time to time and affect your schedule: shortened periods, cancelled classes, holidays, testing days, and so on. Please note these occurrences in the log, as well as any other conflicts. The logs will be collected every two weeks in order that progress and treatment implementation can be monitored.

Summary of Verbal Problem Solving Program

Time: Ten minutes per day, every day

Techniques:

Problems Without Numbers

a) Recast textbook problems so that no numbers appear
b) Prepare these ahead of class time on an overhead transparency, worksheet, chalkboard, etc.
c) Focus on how to solve each problem

Writing Verbal Problems

a) Use graphs, charts, tables from the textbook, newspapers, etc. and have students formulate problems based on these data
b) Use data from situations that arise (field trips, sports, etc.)
c) Have students supply their own data
d) Have students solve each others' problems

Estimating Answers

a) Show students how to estimate
b) Have students estimate orally
c) Estimate answers to text problems before doing computation
d) Eventually have students estimate answers to all verbal problems they work (underline estimate, circle exact answers)

Reading Verbal Problems

a) Focus on word recognition, context, general comprehension
b) Write, pronounce, define new words; give examples and nonexamples of the concept
c) Read problems aloud; use tape recorders
d) Provide reading assistance on an individual basis
e) Have students and teacher alternately read and discuss problems
f) Use text problems, student-created problems, problems from older textbooks

Writing Open Sentences

a) Translate conditions into equations
b) Allow for equivalent equations
c) Use problems from lower grade levels

In many ways, it seemed that the discussion of the program led to some increase in teachers' willingness to implement it, but also perhaps to more variance in the program (in terms of individual teachers' interpretations) than was present in the previous elementary school sample. That is, group discussions at the time appeared to support the notion that the recommended times for each part of the lesson were generalized statements about an average distribution of time over the year, and that individual lessons might deviate from the average (that not all teachers perceived the situation this way can be seen in Appendix 6). Although the investigators occasionally mentioned that balance among the lesson parts was important, the need for adjustment from lesson to lesson was also frequently expressed. We suspected that teachers in this sample would be much more likely to modify the time allotments than elementary school teachers and secondary teachers who subsequently were asked to use the treatment but did not participate in the modification process. (However, subsequent information suggests that the tight time lines presented in the manual and perhaps information given at the training session led teachers to feel that they should not vary time lines.)

Although there were not many alterations in the program, we feel that the changes made were useful and important. In addition to their substantive contributions, teachers who were involved in the discussion may have been very important symbolically to subsequent work with teachers who were asked to use the program. That is, the knowledge that other secondary teachers had examined the program may have been instrumental in obtaining teachers' cooperation and participation and perhaps increased their adherence to program suggestions.

Outside Evaluation

The meeting with the partnership teachers was tape recorded in order to allow for outside evaluation about the conditions and processes of that meeting. We were very fortunate to have the professional consulting services of Dr. Dee Ann Spencer-Hall, a sociologist trained in the qualitative tradition (symbolic interaction) and a former classroom teacher. She is very interested in classroom process and has completed several field studies in classrooms. Although she is a colleague and friend, we were confident that Dr. Spencer-Hall would provide a rigorous critique of the research and bring to the study a perspective which complimented but broadened the perspective of the principal investigators (we are pleased to report that our positive expectations were fulfilled).

Dr. Spencer-Hall listened to the tape and analyzed it. She concluded that teachers had an opportunity to contribute to the modification of the program and were even encouraged to do so. However, she indicated that teachers appeared to be stating beliefs and suggestions which tended to present and support their own teaching practices rather than proactively dealing with the program as a means of developing an approach to mathematics instruction. These were veteran teachers who in some ways seemed to be justifying their current teaching practices more than they were searching for new and integrative approaches. At a minimum, Spencer-Hall felt that the program did develop some common boundaries, set the conditions for minimum participation, and produced a few program modifications (although this varied from teacher to teacher). In subsequent interviews with the teachers in the program (after the program had ended), she found that many teachers were concerned about the number of changes which were occurring in the district (for example, the closing of several schools and the shifting of teachers from one building to another), and she suggested that many of these factors may have led teachers to be more personally oriented than program-oriented during the decision-making meeting.

Context Effects

We were sensitive to possible context effects in the data. Another group of teachers might have different reactions to the program and their own unique suggestions for modifying the program. This is possible for a variety of reasons, including the individual teachers attending the meeting and the nature of the district and its relationship to the community. Although we did not have the time or resources to examine these context effects directly, we did want to examine them indirectly. We wondered if teachers who were making recommendations about a program they would implement would evaluate the program differently from teachers who were reviewing the program but who did not have to use the modified program. One could argue that teachers who will teach the program would become much more interested in it and thoughtful about it. On the other hand, it could be that teachers who do not have to utilize a program (and perhaps have to do extra work) may feel freer to make more recommendations than they would if they had to implement those changes in their own instructional programs.

To consider these context effects, we met in a different city with another group of junior high teachers who knew that they would not be required to use the program. (This meeting took place prior to the meeting with users...the partnership meeting described above.) These teachers were also given both training manuals to read prior to the meeting and were requested to bring their critiques of the program when they attended the meeting. (The critiques of these twelve teachers appear in Appendix 5). The teacher responses indicate that these teachers were very supportive of the program.

After these materials were collected, there was a discussion with the teachers similar to the one reported above. Teachers were asked to help us improve the program and were encouraged to make criticisms and suggestions which would make the program more practical for use in secondary classrooms. These teachers expressed much more positive affect about the program during the meeting than did teachers who would subsequently use the program.[*]

Spencer-Hall attended this meeting with "non-users" as an observer and her reactions to this group of teachers were very similar to our own. She noted that non-users were much more positive in describing the program and showed more interest in its potential value for teaching mathematics. She also noted (as we did) that these teachers were much more energetic, younger, and more knowledgeable (at least more expansive) in their discussion of mathematical concepts and instructional strategies. In general, both the principal investigators and the outside consultant/evaluator felt that this group of non-users would have been much more enthusiastic in their implementation of the program than the other group. However, it is impossible to determine whether our beliefs would have been matched by actual behavior. At a minimum, these data suggest that the modifications suggested by teachers were reasonably consistent across two different samples and that the condition of teaching or not teaching the program did not appear to mediate suggestions (although affect and involvement did vary between the two groups).

As noted above, Spencer-Hall had access to the tape recording of the "user" meeting and had observed the "non-user" meeting. The following section presents her comments about these two groups of teachers (her comments have been condensed and edited; however, she has approved the present version). It should be noted that when Spencer-Hall wrote this report, she had interviewed several of the "user" teachers and hence her comments are based on these interviews as well as on her information about what transpired during the two group meetings.

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[*] In general, the questionnaire reaction of both user and non-user teachers to the treatment program was very favorable. For a comparison of user and non-user responses as well as pre-post changes in user teachers' reactions to the program, see Appendix 5.

Spencer-Hall's Comparison of the Two Samples

> In comparing information from meetings between the non-user and user teachers, I have discussed two types of differences between these groups. First, I think there were some differences in individual characteristics between the two groups of teachers. There were also some differences between the two school districts which influenced teacher participation in the program. I should point out that because I do not have individual data from the non-user teachers, my comments are based on observations of their general meeting in the fall of 1979. (If I had interviewed some of the non-user teachers, my comments would be more definitive.)
>
> The non-user teachers generally appeared to be much younger than the user teachers. I am not suggesting that age differences would have affected teachers' evaluations and use of the program, but that the older user teachers had a more negative attitude toward students and their teaching, which reflected a disenchantment and pessimism developed over many years. Some of the general problems related to teaching which were expressed by non-users were associated more with the general problematic nature of teaching (e.g., filling out papers, managing five or six classes of adolescents a day, etc.); whereas users tended to link teaching problems to a perceived change in students of today. Most of the user teachers thought that students today were out of control for various reasons: were from homes of former hippies; watched too much television; were from "broken" homes; or were directly or indirectly influenced by drugs. This perception of students probably made users' teaching less enjoyable and less effective. Non-users did not blame students and/or parents for classroom problems. As I said, perhaps the older teachers have seen negative change over a longer period of time. The younger teachers may not have such a historical perspective and thus they are not able to compare today's students to students of fifteen years ago.
>
> The two groups of teachers also had contrasting perspectives on change. The non-user teachers were supportive, enthusiastic, and positive about the program because it offered the possibility for change which might make their teaching more effective, and they seemed willing to take steps toward that change. On the other hand, the user teachers seemed more resistant to change. Most of them said they did not alter their teaching at all when they used the program, that it reflected what they had always done. This resistance is of concern for several reasons.
>
> The results of the study were probably affected if user teachers did not really change their instructional programs. (Was what experienced teachers have always done being measured,

or the instructional procedures outlined in the program?) The investigators may also have had some misconceptions about what teacher behaviors should be changed and how. If user teachers' prevailing attitude was that they need not change from their old ways, perhaps they were automatically denying that certain aspects of their behavior did need to be changed. Even when user teachers agreed to participate in the study and were well paid, they ignored the possibility for change.

Most of the user teachers did not take work home; this finding was different from what teachers in my other studies have reported. For example, last week I interviewed a math teacher who estimated that she spent up to thirty hours a week outside class grading papers, etc. Non-users' questionnaire responses would allow comparisons between their outside preparation and that of users.

Comments made at the meeting suggested to me that the morale of user teachers was generally lower than non-users' morale. Differences in conditions between the two school systems may explain this variation. We can safely assume that all school systems are suffering the same structural problems, which are due primarily to economic factors. However, the users' district was experiencing these problems to much greater extent than the non-users' system. The combination of economic problems, declining enrollment, and desegregation had caused disruption to the whole user district, particularly to the junior highs because of school closings. User teachers were thus primarily concerned about whether their particular schools would stay open and if not, what this would mean to their future careers. School closings were particularly threatening to teachers who had about twenty years experience, and had been looking forward to a smooth transition into retirement. I believe this was a dominant concern among the user teachers, one which overrode their commitment to participation in the study. In comparison, non-users worked in a more stable and more affluent school system. Although contact with school administrators in the non-users' district can provide more information about this phenomenon than I can, non-user teachers did not mention school district problems as a concern which affected their commitment to teaching or to the program.

The pervasive organizational-systemic-financial problems and pressures on the user teachers, and their generally negative attitudes towards students and teaching, were predominant factors which probably affected their commitment to the study. The non-user teachers, who were less concerned about these kinds of problems and had a more positive attitude about students and teaching, exhibited a more enthusiastic response to the program.

There were some areas of concern which the two groups of teachers voiced about the program, although I am certain these criticisms are reflected more clearly in their responses to the questionnaire. A major criticism was with the lack of flexibility they perceived in the program. This concern may have been partly due to teachers' interpretation (incorrectly) that they

> could not vary from the time schedule at all in order to fulfill the requirements of the study. Secondary schools follow very different and more rigid time schedules than elementary schools, and secondary teachers found it difficult to implement the program (or predicted it would be difficult to implement) when it did not fit into their ongoing time schedules (this difficulty also reflects a certain rigidity on the part of both users and non-users). Furthermore, at the secondary level class periods are frequently interrupted by other activities: assemblies, class meetings, etc. Investigators can hope that teachers will be flexible enough to work around disruptions, but often they are not. Perhaps a more realistic alternative would be to build flexibility into the math program.
>
> Another common criticism was that the program was not practical for use with lower-level students. This complaint was also voiced[*] by teachers who participated in another study (Keziah, 1980).
>
> In summary, non-user teachers were enthusiastic, supportive, and positive about the program. I should point out, however, that three teachers made most of the comments. Three-fourths of them made some comments, and three teachers said nothing. Although user teachers seemed positive about the program because it was similar to what they had been doing for years, they had a negative attitude about teaching in general and a pervasive concern about the future of their school system and their own positions in the system.
>
> In conclusion, the relative success of the math program is in part a reflection of teachers' attitudes about teaching and their students, and the degree to which their school systems are experiencing difficulties. Implementing the program is not purely a matter of changing instructional procedures. I feel that had the program been implemented in the non-users' district, students' achievement gains would have been greater. Students in districts such as the users', where teacher morale is low and populations are rapidly shifting, are at a disadvantage, even when their teachers utilize the best programs.

An examination of the planning comments which "non-users" were asked to make at the end of the Teaching Style Inventory (teachers brought these to the meeting) indicated that most "non-user" teachers reported that they spent large amounts of time planning at home (all but one teacher spent from 1½ - 2½ hours each night). Hence, this and many other differences between the two teacher groups have been noted.* What effect, if any, such differences would have on program implementation is, of course, impossible to determine. However, such data suggest that the context of the research site in teacher partnership work is very important. Furthermore, the teacher interview data collected by Spencer-Hall and those collected by Good and Confrey (to be described later) will help to illuminate the difficult conditions under which some teachers work. Such data also indicate how those conditions (and teachers' perceptual reactions to them) affect what can and cannot be accomplished through in-service programs.

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[*] Also (as noted earlier) a few sixth-grade teachers cited this as a weakness of the program. However, our analysis of achievement findings do not show systematic problems for low achievers.

Method

Sample

The research took place in a large Southwestern city. The investigators met with school administrators during the summer in order to explain the project and to obtain permission to do the study. School administrators explained the project to principals, who in turn described the project to classroom teachers. All junior high schools in the district were contacted, except for several Title I schools which were participating in another mathematics experiment. The school administration felt that two experiments within the same school would be unwise and we agreed.

In some schools all eighth-grade teachers volunteered for the project; in other schools, only one teacher volunteered. After determining the number of teachers who would be participating in each school, we again discussed the sample with the school administration. We wanted to balance the sample as closely as possible according to the schools that control and treatment classes came from. Because we had to meet with partner teachers early in the year, it was impossible to derive the sample after the administration of the pre-test (the best research strategy). Based upon information the school districts gave us about the student population each school served, we divided the sample into matched sets of schools, and randomly assigned schools to treatment conditions.

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*We also asked our observers to interview user teachers about outside class participation (see Appendix 7). These results also indicate that user teachers did little outside class preparation. Also, the observers wrote a brief sketch of each teacher. These profiles appear in Appendix 8.

Nineteen teachers volunteered for the project. Of these, six were assigned to the partnership group and five teachers were assigned to the treatment group. Both the partnership teachers and the treatment teachers were asked to use the instructional program in their classrooms. The only difference between the two groups was that the partnership group had a chance to modify the program, and the treatment group did not.

Five teachers in the control group allowed us to observe their classrooms. Three other teachers in the control group allowed us to observe and to collect pre- and post-achievement data in their classrooms, but did not attend the orientation training (we called these teachers non-participating controls to distinguish them from the control teachers who received a motivational treatment).

The nineteen teachers were drawn from twelve different junior high schools. Most of the target classrooms in the study were regular eighth-grade classrooms. However, because one partnership teacher who was basically teaching algebra suggested at the partnership meeting that it would be interesting to compare algebra classrooms as well, we did build a minor, pilot algebra study into the design. The distribution of regular eighth-grade classes and eighth-grade algebra and ninth-grade algebra classes across the entire sample was as follows: partnership teachers, ten, one, and three; treatment group teachers, seven, zero, and two; regular control, nine, three, and one; non-participating control, five, zero, and zero. The final sample is summarized in Table 6.

Teacher Training

Two weeks after the initial meeting with the partnership teachers, the project directors met with sixteen of the teachers from the twelve schools, the school principals, and district administrators. At the training workshop, all participants were told that the program was largely based upon earlier observation of relatively effective and in-effective fourth-grade mathematics teachers. Furthermore, we explained that subsequent research in fourth- and sixth-grade classrooms had provided experimental data to illustrate that students in classrooms of teachers who had been exposed to the treatment did better in some areas of achievement than did students in control classrooms. We also told participants that a group of junior high teachers from their own district had been working with us to modify, and hopefully to improve, the program. Teachers were informed that they were going to be requested to teach the modified program.

Teachers were told that although we expected the program to work, the earlier correlational/experimental work had been conducted in elementary schools, and the present project was the first test of the program in secondary schools. After a brief introduction, the teachers and their principals were divided into two groups. Teachers in the treatment group (including both partnership and regular treatment teachers) were given an explanation of the program (the training lasted ninety minutes). After the training session, regular treatment teachers received the 45-page manual, along with instructions to read it and to begin to plan for implementation (partnership teachers already had read the manual). In this manual, definitions and rationales were presented for each part of the lesson, with detailed descriptions of how to implement the teaching ideas. In addition, treatment teachers were also told what modifications were made by the teachers at the planning meeting and also received the verbal problem-solving manual and the more precise procedural directions which had been developed by teachers at the partnership meeting.

Control teachers were told that they would not receive details of the instructional program until later in the year, but that we hoped this information might be especially useful to them then. At that time they would receive information (i.e., other teachers' comments) about the program itself. Finally, we informed control teachers that their immediate role in the project was to continue to instruct in their own styles. Because the control teachers knew that the research was designed to improve student achievement, that the school district was interested in the research, and that they were being observed, we feel reasonably confident that a strong Hawthorne control was created (as noted previously, three control teachers did not attend this orientation meeting).

The partnership teachers were paid $200 for their participation in the project, all other teachers were paid $100. Teacher honorariums were paid for attending workshops and for filling out logs of their teaching activity. At the end of the project, all teachers who agreed to an interview, who provided a critique of the SRA test, and who filled out another teacher belief instrument were paid an additional $100.

Observations

Control and treatment teachers were observed approximately twelve times during the study. Each classroom in the project was observed four to seven times depending on the number of classes the teacher had in the study. If the teacher had three or four classes, then only four or five observations were made in each of the teachers' classes. If the teacher had only one or two classes involved in the study, then each class was observed six times. Each observer conducted approximately one-half of the observations in each of the classes and observations in each class were equally spaced throughout the duration of the study.

All observations were made by two doctoral students in mathematics education who were living in the target city during the project. The observers were trained initially with written transcripts and videotapes, and then in actual classrooms. Observers reached reliabilities of .80 on each of the coding distinctions used in the actual study. The classroom observational form, the observational checklist, and the content logs filled out by control and treatment teachers (biweekly) can be found in Appendix 9.

Schedule for Meetings and Testing

The initial meeting with partnership teachers took place in late September and the training/briefing session with all the teachers was held during the first week in October. Pre-tests were administered in the second week of October and classroom observations began shortly thereafter. The post-test was administered in January; hence, the treatment lasted about three months. The mathematics pre-test had been used in the Texas Effectiveness Project and was provided to us by Dr. Carolyn Evertson. The post-achievement tests were two subtests of the SRA, Level F, Form 1 test (Math Concepts and Problem Solving). Reliabilities on both instruments are excellent. These instruments can be found in Appendix 10. The Aptitude/Attitude Inventory used was the same instrument we had used in our previous work in elementary schools (for detailed information on this instrument, see Ebmeier, 1978). This instrument can be found in Appendix 11. Also in Appendix 11 is the Teaching Style Inventory that was used to assess teachers' beliefs about mathematics.

Program Implementation

Teacher opinions. After terminating the project, we wanted to obtain teachers' perceptions of their involvement in the program. We felt that it would be useful to have someone else collect these data for us, and as reported earlier, Dr. Dee Ann Spencer-Hall was willing to do this. She traveled to the target city about a month after the project had been concluded, and interviewed all six of the partnership teachers and three of the five treatment teachers (she also interviewed two of the control teachers, but those data are not relevant to the present discussion).

The interviews lasted about one hour; a few of them had to be conducted via phone (because of an ice storm). Dr. Spencer-Hall described herself as a former teacher and as a sociologist interested in ideas people have about teaching. She indicated that although she wanted to ask several questions about the project, she was not one of the project staff and simply wanted to know what their reactions were to the program. She stated that criticisms were welcome.

At the conclusion of her interview work, she drafted a brief summary for the project directors. A very condensed and edited version of her comments (which has been read and approved by her) follows:

> In retrospect, I think the interviews went quite well. Most of the teachers appeared to be open and honest with me in their responses. I plan to give you feedback in two ways. First, I have enclosed the interview schedule, the questionnaire I used (with modifications depending on which category a teacher was in), summaries of each teacher's interview, and my own subjective reaction to the interviews. Second, I am having the tapes transcribed because I feel that many comments were worth having verbatim and in their entirety.
>
> At this point, I would like to make a few subjective comments. My first reaction to the teachers was that they were all very experienced. For example, five of the eleven teachers have taught twenty years, two have taught over fifteen, and the remaining four have taught over twelve years. The range of twelve-twenty years, with the mode of twenty years gives you an interesting sample. I realize you will get this data from the questionnaire you had them fill out, but the thing that struck me in

the interviews was that (a) three of the five who had taught twenty years were obviously burned out on teaching, and (b) four of these five were sure that kids today have "gone to the dogs." (Although some in the over fifteen year range expressed the same attitude.) My opinion is that such dissatisfaction with teaching and the attitude that kids today are disturbed, of low ability, and generally hyper and out of control affects these teachers' effectiveness--and ultimately, in this case, reduced their enthusiasm for trying something new, i.e., your program.

A more important consideration, however, also seems to be related to the large number of years of experience. Most of the teachers felt that because of their many years of experience they had learned or developed the most effective way of teaching. This attitude was also reflected in the fact that six of them said they did absolutely no preparation outside of class, three did only fifteen-thirty minutes, and two spent over one-and-a-half hours a night. Those who did none were even incredulous (they all laughed hilariously) that I would suggest that they had to prepare. I suppose their reasoning was that anyone who has taught the same subject for fifteen or twenty years at the same level must be completely incompetent if he/she has to prepare for classes. Because of the pervasive attitude that they were teaching math in the most effective way possible, the program may have done little to change their behavior.

In summary, as I said before, these comments are subjective and only somewhat systematic at this point; they only relate to general areas and concerns. The interviews, however, did reinforce and substantiate my own opinion that the data should be contextualized. Even though the data are primarily quantitative, your methodological strategies have taken you into the qualitative area, through partnership information discussions and my open-ended interviews, and thus more consideration should be taken of the contextual features of day-to-day life in the schools and classrooms. For example, what about the context of a sixth-grade class is different from an eighth-grade class and would make implementation of the program more problematic for eighth-grade teachers? The situational context in a junior high is probably more disruptive, more complex (due to size, for example, and having six or seven different classes), has more conflicts, and is less predictable (in terms of events and individual behaviors). This would partly explain the teachers' dissatisfaction with the routinized nature of the program; it does not realistically reflect the "reality" of the junior high world. To reiterate my opinion, classroom events occur in complex, problematic situations and in contexts which impinge and sometimes inhibit classroom routines. Cannot these contextual factors be integrated into discussions and even into the programmatic aspects of teaching math? (The result would surely be a book.) I will save further comments for later and will look forward to any reactions from you and to future discussion. I am thoroughly enjoying my participation.

Brief summaries of partner and treatment teachers' reactions in the Spencer-Hall interviews are presented in Appendix 6. However, these are only a small portion of the total interviews. The interview schedule she used included the following questions:

I. School

1. What is it like to teach in this school?
2. What kinds of things *inhibit* what you do in your classrooms?...Problems which interrupt your work?

II. Teaching

1. How do you feel about teaching, i.e., how satisfied are you with teaching as an occupation? (Probe)
2. What things might happen to increase your satisfaction?
3. What bothers you *most-least* about teaching?
4. Do you think teaching math is any different from teaching any other subject? (How--in what ways?)
5. Do you teach differently in different math classes; in the same course at different hours of the day? (Why--How?)

III. Math Effectiveness Program

1. To what extent do you feel you've become involved in the program?
2. Did you change anything about the way that you've taught math as compared to past years? (What--How?)
3. Do you see this as a positive change--is your teaching more (or less) effective now?
4. What do you see as the major strengths of the program?
5. Weaknesses? (Why)
6. If you could change the program in any way(s) what would you do? (Why) Additions--Deletions?
7. Would you (continue) to use it if changes were made?
8. Did you communicate with other teachers about the program? (How, what happened?)

IV. Relationship to Researchers

1. Did you feel you had adequate input into the program during your early discussion with the researchers?
2. Did you feel you had an impact...that your comments and suggestions were taken seriously?
3. What (if anything) could have been done to have made communication with the researchers more effective or helpful?
4. What is your *general* feeling about classroom teachers and researchers working together on instructional projects? (Probe)

V. Outside School

1. What do you do when you're not at school?
2. How much of your time outside school is spent in planning for your time inside school?
3. Did participation in this program change your planning time outside school? (How?)

We are presently analyzing these interviews to determine teachers' beliefs about general aspects of teaching, and to see if these beliefs can be related in any way to effects on students. These analyses were delayed because we anticipated collecting interviews which were more specifically focused upon the teaching of mathematics (these were collected and will be discussed later). We did not want to be influenced by previous knowledge about teachers (previous interview responses; knowledge about teachers' effects upon student achievement; etc.) when these interviews were conducted.

Our own analysis of the teacher interview responses to questions about program implementation is that the responses approximate a bell-shaped curve. One partnership teacher (01) had especially strong negative feelings. This teacher reported that the program had no effect upon her/his behavior and that he/she felt coerced into participating in the experiment. The observers felt that the teacher implemented the program at a minimum level despite these reactions. Another teacher (03) reported compliance with certain aspects of the program, but observers found virtually no involvement and/or participation in the study. Other teacher comments about the program in general were reflected in observational data (to be discussed later). For example, one teacher (02) who liked the program because of the verbal problem-solving strategies did emphasize verbal problem solving in the classroom.

Four partnership teachers reported that they liked the meeting and that the introductory meeting with researchers and the subsequent training session went well. Two of those who approved of the program pointed out specific parts that were good; the other two emphasized the similarity between their own teaching and the present program. Similarly, the treatment group had a mixed reaction to the program and to the research training session (they did not participate in the modification session).

Both groups of teachers reported that the program was similar to what they were already doing and indeed, Drs. Lanier and Evertson had mentioned this possibility to us previously, suggesting that it was both an advantage and a disadvantage. It was an advantage in the sense that teachers would be willing to implement the program and to make some modifications in their behavior. However, teachers might not be motivated to look for some of the subtle (but important) differences between the program and their present teaching methods.

One year after the project ended, fifteen of the participating teachers were interviewed (to be described later). Although teachers were not explicitly asked about their reactions to the treatment program, all treatment teachers initiated on their own some comments about a particular part of the program which was meaningful to them. Some teachers said that though they had been conducting class review sessions in the past, the program had prompted them to think systematically about the nature of review and how they could build broader reviews into the lesson. Other teachers indicated that the program had helped them to consider instruction in problem solving and to emphasize this topic more than they had. Some participants commented that their general approach to development and seatwork assignments was very similar to the approach advocated by the program, but that they were now much more careful to be certain that students were ready for seatwork before they assigned it. In general, teachers did not make comments about the program as a whole; rather, they chose to comment upon particular parts which seemed especially meaningful to them. We suspect that teachers were surprised by the general similarities between the experimental program and techniques they had already been using in the classroom. The contrast between their expectations for the program and the actual program may have made the similarities (and hence their reports of them) highly salient. However, their behavior during the program (as we shall see below) and their comments at the end-of-project interviews suggest that some teachers were noting and responding to subtle differences between their teaching practices and program recommendations, at least in some areas of the program.

Observer research. An initial step in analyzing data was to determine the extent to which partner and treatment teachers had implemented the program. Because participants reported that the program was similar to instructional methods they were already using, it was important to determine whether treatment teachers and control teachers differed in any systematic ways in their classroom behavior (i.e., was the treatment condition associated with some distinctive teaching behavior?).

Table 7 presents selected implementation data for all project teachers. The first six teachers are partnership treatment teachers. Teachers seven through eleven are regular treatment teachers, who were asked to implement the program and were trained to implement the program, but did not have a chance to modify the program. Teachers twelve through sixteen were control teachers who attended the orientation meeting and who were observed. As noted before, there were three other teachers who served as controls, and who were observed but who did not attend the orientation meeting (these control teachers were called non-participating).

Our first task was to examine the implementation scores to determine if partner and regular treatment teachers were using the program. From just the variables presented in Table 7, it is clear that some teachers implemented the program more fully than did others. The average implementation score (more on this below) is based upon the five variables presented in the Table, as well as several other variables (including the assignment and checking of homework, the presence of controlled practice, the presence and quality of review work, etc.).

The information presented in Table 7 allows the reader to see the variability between and within treatment and control teachers on selected aspects of the program.

A general implementation score was assigned to each teacher at the end of each observation. The score assigned to the teacher was based on the following scale:

(5) If the teacher implemented all major components;

(4) If the teacher implemented most of the major components;

(3) If the teacher implemented about one-half of the program components;

(2) If the teacher implemented some of the program components;

(1) If the teacher implemented very little of the program.

Innercoder reliability was estimated on the implementation scores on the basis of seven dual observations made in the target school district during the first two weeks of the study. Perfect agreement was found on five of the seven observations and only a one point variation found for each of the other two observations.

At the end of the study, a comprehensive implementation score was determined for each teacher, by averaging on the individual implementation score assigned to the teacher at the end of each visit. These means are reported in Table 7.

One important question to raise is, does degree of program implementation correlate with residual gain in computation, problem solving, and attitudes toward mathematics? To answer this question teachers' average implementation scores, average time spent on mental computational instruction activities, and average time spent on verbal problem-solving activities were correlated with students' residual scores for computation, problem solving, and attitude. These results are presented in Table 8.

As can be seen in Table 8, the average implementation score does not correlate significantly with students' residual performance on the computational or problem-solving test. However, average implementation score was found to correlate significantly with students' attitudes toward mathematics ($p = .02$).

The correlation of instructional time spent on mental computational and instructional time spent on verbal problem-solving activities are also presented in Table 8. These parts of the program were computed separately because they were presumed to be relatively novel instructional acts (not frequently engaged in during secondary mathematics instruction).

As can be seen in Table 8, the average time spent on mental computational activities did correlate significantly with students' problem-solving residual scores ($p = .05$), but not with their computational scores nor attitudes toward mathematics. Interestingly, instructional time spent on verbal problem-solving activities did correlate significantly with students' residual scores for computation ($p = .05$), problem solving ($p = .02$), and virtually reached a significant relationship with students' attitudes toward mathematics ($p = .09$).

An examination of Table 7 shows that partner and regular teachers were found to implement some aspects of the program more often than did control teachers. The mental computation and verbal problem-solving activities differentiated the two samples most sharply, and there was considerable overlap between treatment and control teachers in terms of the amount of time spent on development and seatwork, and in the overall quality of the development lesson. Overall, these data suggested that somewhat different behaviors were occurring in treatment and control classrooms and hence, an analysis of achievement effects would be meaningful (especially in the area of problem solving).

After examining the implementation data, we decided to eliminate teacher three from subsequent data analyses because this teacher was generally deficient in implementing the program. In particular, this teacher did not utilize the mental computation activities and virtually ignored verbal problem solving throughout the course of the experiment.

Observer opinions. At the conclusion of the project we asked the two observers to describe briefly each teacher they had observed and to provide their general impressions of the teachers, their effectiveness, and the extent of their program implementation. The two observers' comments appear in Appendix 8. In general, their comments reflect important variations among teachers in the extent to which features of the program were present in control and treatment classrooms. Observers' comments also generally support the implementation data derived from actual observational records.

All in all, the implementation data from three perspectives suggest that treatment teachers generally saw the program as quite similar to teaching techniques they had been using previously. However, there is evidence that treatment teachers were influenced by the program, and that they did instruct in a manner which differed in some ways from control teachers. The data further suggest that teachers were more influenced by certain aspects of the program rather than the treatment as a whole. The verbal problem-solving techniques and mental computational activities appear to have had most impact upon treatment teachers.

Results

After deciding to eliminate teacher three from all analyses dealing with program effects on students (because of this teacher's low implementation score), two general questions affecting the analysis of the data were considered. First, we wanted to know if the algebra teachers in the treatment and control conditions differed in any way (it was assumed that algebra students would do better than non-algebra students on the post-test). This analysis showed no differences between the algebra teachers in the treatment and control classrooms (the p value for post-computation was .71; the p value for post-problem-solving was .34; and the p value for post-attitude was .80). Next we wanted to determine whether the partnership teachers differed from regular treatment teachers, and if these groups should be analyzed separately. These comparisons suggested that there were no significant differences between partnership and regular treatment teachers. The respective p values were: post-computation, .50; post-problem-solving, $p = .91$; and post-attitude, $p = .42$. The effects of partnership and treatment teachers were not different.

As can be seen in Table 9, the pre-achievement level of students in the control group was somewhat higher than achievement levels for treatment classes. Despite this, raw scores of students in the treatment group on both the post-computation and the post-problem-solving sections of the SRA achievement test were somewhat higher than scores of students in control classrooms. Analysis of covariance procedures (using the prescores as a covariate) were performed on the adjusted means shown in Table 9. In performing these analyses, the classroom was used as the unit of analysis and each class that a control or treatment teacher taught was included as a separate unit in the analysis (N = 39). The results of these ANCOVA analyses indicated a weak trend in favor of the treatment group on the computational performance of students (p = .15). However, the effect of the treatment on problem-solving scores of students was significant (p = .03). These results are consistent with the implementation data reported earlier. That is, there was not much variability in general behavior between treatment and control teachers; however, there were noticeable differences in the use of mental computation activities and verbal problem-solving activities.

Follow-Up Teacher Interviews

One year after the formal project had ended, arrangements were made to interview available classroom teachers. Sixteen of the teachers who had participated in the project were still teaching in the school district, and it was possible to interview fifteen of them.

Dr. Dee Ann Spencer-Hall had interviewed many of the teachers when the project concluded; her questions focused upon teachers' reactions to the project and their general beliefs about teaching and schooling. The present interviews dealt specifically with the teaching of mathematics and teachers' beliefs about mathematics teaching. In combination, the two sets of interviews should provide an integrated picture of the beliefs that these secondary teachers had about teaching generally and about mathematics teaching.

In conducting these interviews, we were fortunate to have the professional consulting services of Dr. Jere Confrey at Michigan State University. Dr. Confrey is a former secondary mathematics teacher and is a specialist in mathematics content at the secondary level. She has also done extensive field work involving methods of interviewing students and teachers. She is presently working with Dr. Perry Lanier and others at Michigan State University on an intensive study supported by funds from the National Science

Foundation. Their research examines students' experiences (and their conceptualizations of mathematics content being taught to them) in junior high general mathematics classes. Dr. Confrey assisted both in the conceptualization of the interviews and in the data collection. Some of the interview questions resulted from the National Science project at Michigan State University. However, many of the questions raised were developed specifically for our own research project.

All of the individual follow-up interviews were conducted by Drs. Confrey and Good. Some of the teachers were interviewed jointly so that we could experiment with the interview format. The questions presented in the interview follow:

> What subjects do you presently teach (number of sections of eighth-grade math/algebra)?
>
> When and why did you decide to become a mathematics teacher?
>
> What is the difference between seventh- and eighth-grade math content (the difference between eighth- and ninth-grade math content)?
>
> List the various units that are taught in eighth-grade math.
>
> Do you spend the same amount of time on all units (why or why not)?
>
> Why is math taught in schools? Why, in particular, is eighth-grade math taught?
>
> What's the most important content that you teach?
>
> If students just learned two or three things, what things would you want them to learn in your class? (Does it differ for different students or groups?)
>
> Which content do you spend most time on? Is this because it's hard? Important? Or prerequisite for other material?
>
> How do you decide when to move from one unit to another? Do you ever re-teach lessons? How often? Why?
>
> How much flexibility do you have in choice of content and pace of instruction? Does the district mandate the content of a curriculum?
>
> How much do you know about what other teachers do in eighth-grade mathematics? (In your building...across the district).

Are there some areas of disagreement concerning content or teaching style?

What makes students work hard in mathematics?

How important are grades in motivating students?

On what basis do you assign grades? (Effort vs. performance; the role of tests, homework, and behavior)?

How much stress do you place on application? More for some students than for others? Which ones?

How important is memorization vs. understanding mathematics? Is it more important for some students than for others? How do you define memorization and understanding?

Do all students do equally well in math? Why not?

From your experience, are the highest-achieving and lowest-achieving students (say, the top and bottom one-fourth of your class) closer or farther apart at the end of the year in terms of their mathematics skills? Why? How do you feel about this? If you wanted to change it, what could be done?

What percent of your eighth-grade students are capable of mastering the basic curriculum?

How do you teach the concept of ratio? How do you teach multiplication of decimals? Take the problem .7 x .11 and explain. In working this problem, if a student asked, "But I thought when you times something, the answer gets bigger," how would you respond?

If there was sufficient time in the interview, the following questions were asked:

In elementary school, the subject of reading is often taught in groups, but math is more typically taught to the whole class--why does this happen? How do you feel about it?

Most teachers say that they have good teaching days, so-so teaching days, and bad teaching days. What's your opinion about this? What makes a good day? Has this changed over your teaching career?

The following questions were asked of all interviewees. These questions were taken from the general mathematics project being directed at Michigan State University by Dr. Perry Lanier.

- Mark each statement either true or false.

_____ In mathematics, there can never be more than one right answer.

_____ Problems without answers exist in mathematics; not just problems that no one has answered yet.

_____ An answer in mathematics is always either right or wrong.

_____ If there were no people in the world, mathematics would still exist.

- If you had a choice, which of the following would you prefer when solving a problem:

_____ to have one method which works in all cases.

_____ to have more than one method which works in all cases.

_____ to have more than one method which works in some cases.

Why?

- Is it possible to get a right answer to a problem in mathematics and still not understand the problem? Explain. Is it more true of some students than others? Which ones--what percent of your students is this true of?

- Is it possible to understand a problem and still not be able to get the right answer? Explain.

- Below are eight statements which describe mathematics. Rank the statements which best describe mathematics from 1 (best describes math) to 8 (least describes math).

Rank

1. Mathematics is like a bag of tricks. ____

2. Mathematics is like building a model airplane. ____

3. Mathematics is like playing the lottery. ____

4. Mathematics is like doing chores. ____

5. Mathematics is like following a recipe. ____

6. Mathematics is like composing a song. ____

7. Mathematics is like flirting with a sweetheart. ____

8. Mathematics is like telling the truth. ____

- Check the following activities which you consider as part of mathematics.

_____ Doing fractions.

_____ Constructing a jigsaw puzzle.

_____ Changing decimals to percents.

_____ Thinking about infinity.

_____ Making change for dollar bills.

_____ Thinking through a verbal puzzle or riddle.

_____ Finding areas.

_____ Playing chess.

_____ Balancing a checkbook.

_____ Thinking about space.

_____ Solving equations.

_____ Computer programming.

_____ Calculating statistics.

_____ Handing in homework.

These data were collected in the spring of 1981 and the typed transcripts are now being analyzed. We expect to integrate these analyses with other data sources in the project. As mentioned previously in the text, we will include these results in a monograph now being written by Drs. Good, Grouws, and Ebmeier.

Appropriateness of Student Achievement Measure

As a result of the teacher interviews it was possible to examine the adequacy of the achievement test from the perspective of classroom teachers.

The reliability of the SRA (Level F, Form 1) mathematics test is excellent; however, one can still raise questions about validity. We wondered to what degree the test overlapped with the material teachers taught in class and the types of problems presented in textbooks. We attempted to obtain this information in two different ways.

First, we asked Dr. Jere Confrey to conduct a content analysis to compare the SRA test with the *Holt School Mathematics* text (the book most used in the sample). Her interesting comparisons introduced a number of questions about the adequacy of the test in certain areas (especially general computation). Dr. Confrey's complete report appears in Appendix 12. She raises some important concerns about the problem-solving test and the text (the test is a narrower definition of problem solving), but concludes that there is a reasonable congruence between the general text and test (the largest discrepancy occurs between the test and the chapter in the book devoted to problem solving). Her comments raise a number of interesting issues about matching text content and test problems.

When the fifteen teachers were interviewed a year after the project ended, they were given a copy of the SRA test and were asked to critique each test item (Was it taught? Was the question asked appropriately?). All teachers subsequently returned the test ratings by mail. Their responses generally indicated satisfaction with most of the test items, with only two teachers (one treatment, one control) having strong negative reactions to the test.

Although problems exist with the match between test and instruction, the test appeared at least minimally adequate for making comparisons. Also, the test appeared to be equally appropriate for treatment and control teachers (as indicated by teacher responses and by our examination of teacher logs).

Research in Progress

We are still analyzing some of the data collected in the project. As noted previously, interviews with project teachers focused upon their general reactions to teaching as well as their specific reactions to mathematics teaching. We plan to relate teachers' general beliefs about teaching and specific beliefs about mathematics to their classroom behavior and to the achievement gains of students. The second set of interviews was collected in March of 1981; the interpretation of these data is under way and will be reported in Good, Grouws and Ebmeier (in progress).

In addition to teacher interview data, we also have fourth-, sixth-, and eighth-grade teachers' responses to the same 73-item Teaching Style Inventory (Appendix 11). This comparison across grade levels should help us to determine whether teachers' general beliefs about mathematics teaching practices are influenced by grade level and by experience or educational background.

Furthermore, we also have data from large samples of fourth-, sixth-, and eighth-grade students on the same 61-item mathematics Aptitude/Attitude Inventory (Appendix 11). This comparison of students at different grade levels should yield a meaningful profile of the extent (and type) of change in students' beliefs about, and preferences for, certain mathematical practices. These data will also be presented in Good, Grouws, and Ebmeier (in progress).

Although not a formal part of this project, two dissertations were completed that have some relevancy to the work reported here and these studies were supported with a small amount of project money. These studies were completed by the two observers in the project, John Engelhardt and Ruthanne Harre. Brief accounts of their work can be found in Appendices 13 and 14 and extended discussions of their research are in their dissertations (Engelhardt, 1980; Harre, 1980).

Englehardt's research was conducted in sixth-grade classrooms using only the verbal problem-solving treatment. His data suggest that this treatment when used without the larger and more general Missouri Mathematics Program does not have powerful effects on students' verbal problem-solving skills.

Harre's research was conducted in eighth-grade mathematics classrooms and examined the time-on-task behavior of students in treatment and control classrooms. Her data show that students in treatment classes were coded for more apparent involvement than were students in control groups. These data suggest that part of the general effectiveness of the Missouri Mathematics Program may be because it enhances student attention.

Discussion

Previous Research: Elementary Schools

The research presented in this final report is based on two research programs supported by previous grants from the National Institute of Education. The earlier research influenced the present study in substantial ways; it therefore seems appropriate to preface the discussion of the present study with a brief explanation of the context of previous research.

Because of the failure of both educational research and general intervention strategies (not based upon research) to generate meaningful understandings of classroom practice, we decided to *observe* teachers who were *making a difference* in student achievement (students' *mean* residual achievement gain) in a particular *context* (fourth-grade mathematics). We felt that meaningful variation in teaching behavior did occur and we wanted to test this notion, as well as our general belief that individual teachers make a difference in student learning. Our original intention was not to build a comprehensive mathematics program, but to test the hypothesis that teachers affect student learning.

We chose a standardized achievement test as an operational definition of teacher effectiveness. Although this is not a complete definition of teacher effectiveness (or even an adequate definition), we do feel that it is one aspect of teaching which is important. Standardized achievement scores can be a partial criterion if one understands their limitations and does not over-generalize findings based upon them.

The initial study provided evidence that stable and relatively high and low "effective" teachers could be identified, although many teachers fluctuated from year to year in their "effectiveness" (as measured and estimated by the mean residual gain of their students on a standardized achievement test). From behavioral observation of high and low teachers, it was possible to identify patterns of teaching that differentiated these two groups of teachers. These findings were supported by research elsewhere in field settings and by previous experimental research in mathematics education, as well as by observers' comments about instructional variables in the naturalistic study. These findings were ultimately integrated into a program for training and research purposes.

In our first experimental study, we found that fourth-grade teachers were able to implement the program after minimal training (some trouble was experienced in the _development_ portion of the lesson) and that implementation was associated with student achievement gains. Because the differences in test scores between treatment and control classrooms were large, and because the actual treatment was only two-and-a-half months in duration (hence, relatively cost effective), the results suggest that the program is a reasonable method of mathematics instruction. Also, the results (both on the standardized test and the test designed to match the content teachers actually presented) clearly show an important _teacher effect_ in these inner-city school classrooms, which suggests that successful educational interventions are possible. However, it was found that the treatment program was better for some combinations of students and teachers than for others (achievement of all combinations was higher in treatment conditions, but this effect was large only for some combinations).

In the second experimental study, we developed a problem-solving strategy designed to improve students' ability to work verbal problems which appeared in elementary textbooks. Although the project staff felt that this definition of problem solving was a very limited one, we did accept the fact that this was the problem-solving curriculum for many students and teachers. We thus built a program that was designed to affect this type of mathematics performance.

In developing the program, we found virtually nothing in the literature to describe what teachers' views are of problem solving and what they *do* when they teach such content. It is important that future research naturalistically study teachers during problem-solving instruction, to determine whether some teachers are more adept at such instruction than others. Unfortunately, in this project we did not have the time or resources to do this important exploratory work. Instead, we built a program based upon recommendations that were available in the literature and we integrated this advice with our own thinking.

We tested this new training manual in combination with a manual that had been developed in the previous experiment (in fourth-grade classrooms) in sixth-grade classrooms in the same school district. Pre- and post-tests indicated that the program had a significant effect upon treatment students' verbal problem-solving skills. However, the program did not have a significant effect upon general mathematics achievement; probably because the general manual was commonly available in *all schools* (after the successful field experiment in grade four, all fourth-grade teachers in the district were given inservice training in the program). In addition to this possibility of general "contamination," comments made by some control teachers during debriefing procedures indicated that they were familiar with parts of the general program that are not routinely found in elementary school curricula (e.g., mental computations).

Finally, it should be noted that the success of the treatment in the second field experiment was moderated by the form of general administrative organization of mathematics teaching (math as special subject, semi-departmentalized, or open plans). The results of the program are therefore somewhat dependent upon teacher type, student type, and administrative organization, as well as on the treatment *per se*.

The combined results of our work in elementary schools suggested that meaningful improvement in students' mathematics achievement was possible and that the programs which we had developed were reasonable intervention strategies, at least under certain conditions. With this experience, we moved into secondary schools.

Treatment Effects: Secondary Level

The data collected in the project indicate that change in teacher behavior and in student performance in secondary schools is possible. In particular, the results demonstrate that participation in the treatment program was associated with a significant positive effect upon students' problem-solving skills, as measured by the SRA test. Although neither of us thinks that the problems in this test are a completely adequate measure of problem solving, they do represent some skills that appear to be important. It is therefore edifying to see that treatment teachers had a positive effect upon student performance on the problem-solving subtest.

Although we have raised some questions concerning the adequacy of the content criterion test and the level of teacher implementation, the overall evidence suggests that treatment teachers did implement more problem-solving strategies than did control teachers and that the test was a reasonable measure of content being presented in classrooms. We can thus confidently say that the program had a positive effect on treatment teachers' implementation and students' performance on the test.

The data also reflect a moderate, positive trend favoring treatment students' post performance on the computational subtest of the SRA inventory. However, because the difference is minor, it is probably appropriate to state that the training program had no notable impact upon this type of student achievement. This appears to be the case because both treatment and control teachers taught in similar ways during the developmental portion of their lessons. The qualitative ratings which observers made of all teachers are not high and indicate that much future research needs to focus upon conceptualizing and implementing the development stage of the lesson. Although some teachers in treatment and control classrooms were able to conduct development successfully, most qualitative ratings of implementation were not uniformly high.

The teacher interview data are still being analyzed, and from these data we may be able to make statements about the relationship between teacher beliefs and teaching performance, and their effects upon students. These results will also lead to a fuller understanding of the perceptions that secondary teachers have about teaching mathematics and the conditions under which they teach. In particular, interview data will make us more sensitive to some of the difficulties involved in attempts to change teacher perceptions and behavior and will also make us more aware of the difficult circumstances in which some teachers teach. The principal investigators are presently working on a book with Howard Ebmeier (Good, Grouws, and Ebmeier, in progress) and more extensive information about the teacher interview data (the general interviews conducted by Dr. Dee Ann Spencer-Hall and the interviews which focused specifically upon mathematics teaching, conducted by Drs. Tom Good and Jere Confrey) will be discussed in this publication.

Working With Teachers

We began our program of research several years ago by observing what elementary teachers (who were more and less successful in obtaining student achievement) did in the classroom and by building a training program that was sensitive to those differences (although it included a few other components as well). In the present research we attempted to adapt this program for use in secondary schools by working with secondary teachers.

We found that the opportunity to work with teachers to modify our program was an interesting and valuable experience. In retrospect, we would have done some things differently. In particular, we now feel that it would have been more appropriate to have spent considerably more time on procedural aspects of the study than we did (e.g., what the observations were for, how they would be used, when results would be provided to teachers, etc.). More time spent on procedural and social interactions before initiating a focused, decision-making discussion with teachers would have been advantageous. Some initial formal contact with teachers should take place before any substantive discussion, and such meetings should probably give teachers more information about the research and lessen some of their personal concerns about observation and involvement. Such procedural orientation should take place before teachers are asked to read the manual; later, when they have read the manual, they might critique it more fully on substantive grounds.

Another procedure which we would modify involves the initial contact with teachers. Because of the geographical distance involved in this study, this initial contact was made by school administrators. We made some assumptions about the amount of procedural information teachers possessed; however, we found that some teachers did not receive all the information that we thought had been communicated. Also, when working with volunteer teachers, it would be very helpful for investigators to contact teachers and to allow teachers to agree (or disagree) about participation and to negotiate general collaborative arrangements. Some teachers appeared to have volunteered, but without an affective commitment to participation. Investigators could meet with a large group of potential teachers, engage in social and procedural interactions, and only then allow teachers to make a decision about their participation.

In retrospect, the present treatment/discussion meeting may have been too demanding. That is, the teachers were requested to comment upon all aspects of the program at a single meeting. If we were repeating the study, we would instead hold two or three different meetings. At the first meeting only development and work on modifications of that aspect of the program would be discussed. In the second meeting seatwork, review, and homework program components would be considered. Such arrangements might lead to a more focused and more thorough evaluation of the program. If problems developed, it might be useful to have yet a third, follow-up meeting to resolve some issues. For example, in the present study, teachers in general had strong feelings that the testing procedures should be systematized, but they had widely varying ideas about how to do this. In retrospect, it seems appropriate to form teacher committees which attempt to develop tentative solutions. Such committees could bring their work back to the whole group for discussion, review, and modification.

Many other changes are also possible in the partnership arrangements; however, the wisdom and desirability of additional strategies depend upon the particular problem being investigated and the types of generalizations which teachers and researchers are trying to make. If investigators are trying to get maximum teacher involvement, it probably is important to bring in videotapes of teaching (particularly tapes which focus on the development portion of the lesson) and to allow teachers to jointly critique and review program components. Teachers could also observe and be observed by fellow teachers in the partnership group so that they could develop fuller understandings of the program and improved strategies for implementing it.

As the data indicate, teacher reactions are likely to vary from site to site (for a variety of reasons), and one critical factor that would have to be considered in any partnership work is how to balance the many demands teachers already face with new demands imposed by project participation. Some teachers would appreciate increased opportunities to interact with other teachers and to discuss the program; however, other teachers may react negatively to extended or involved arrangements. Indeed, it is important for these procedural issues to be resolved by researchers and teachers jointly at their initial meeting so that a common set of expectations about time required for participation (as well as the form of such participation) could be developed.

Because of the context in which we worked (especially time constraints) and the limited amount of time we spent meeting with teachers, we are reasonbly pleased with the level of teacher involvement obtained and the ideas which were incorporated into the program. Several program modifications were made, and we think that these changes were appropriate and important for adapting the program to secondary schools. These ideas were essentially teacher-initiated, and we are grateful to the participating teachers for their input and assistance. Teachers' brief involvement in training did appear to alter certain aspects of some teachers' behavior, and increased their involvement in the project. However, project involvement did not have a positive effect on other teachers. These individual variations among teachers are similar to results reported by Ebmeier (1978) in the elementary school project. Some teachers in that study implemented the program more fully than others. In particular, Ebmeier found that program implementation was higher among teachers who felt that they were already teaching in ways recommended by the program, and by teachers who were searching for new alternatives. The brief partnership and the general training procedures which we utilized in this study would appear satisfactory for obtaining program implementation from such teachers. However, for teachers who are not interested in seeking alternative solutions and who feel that the program is contrary to their teaching styles, more elaborate procedures and more time will probably be necessary.

Future Research

Development lesson. In our work in elementary schools we found that many teachers do not regularly use an *extended* development component in their mathematics lessons. The treatment appeared to be helpful in elementary schools because it increased the amount of time elementary school teachers were utilizing for development, and it thus helped them to become more active in their teaching of mathematics. However, we found that most secondary teachers regularly include a development portion in their lessons and that time, per se, is not as important as is the *quality* of development. If improvements are to be made in teachers' instruction during development, it seems important to generate more adequate procedures for conveying to teachers criteria which can be used to estimate the quality of the development phase of their lessons. As a beginning step, we are making some films so that teaching and training in development can be more accurate and more specific in the future.

More *content-focused development* needs to be emphasized in future research efforts. Although the program provides general strategies for teaching mathematics, particular content needs to be studied more thoroughly. Better conceptualization of the instructional demands of different types of mathematical content is needed and information about how the development portion of the lesson can be adjusted in ways that are consistent with changes in content. Although the purpose of the program is not to develop generic lesson plans, it is designed to encourage careful thinking and analysis by individual teachers. In its present form, the program does not do enough to promote critical thought about different types of mathematics content or about which strategies are more or less appropriate for teaching different types of content. Time allocations suggested in the program should probably vary with different types of mathematics content, as well as according to the lesson stage. The same sensitivity to variation should be built into the observational coding system and checklists which are used for classroom observation. Both the program's instructional strategies and observational procedures need to be more closely related to content issues in the teaching of mathematics.

Teacher variation. We began the program of research about a decade ago in order to answer the question, Do individual teachers make a difference in students' learning during mathematics instruction? It was not our intention to build a perfect mathematics program, but rather, to determine whether teachers had an effect on students' achievement and/or attitudes. Using strategies derived from naturalistic observation of successful elementary teachers, we wanted to see if other teachers could use these behaviors in their own teaching and to estimate what impact, if any, these behaviors had on student achievement. In our subsequent work we have become more interested in improving students' mathematics performance and in helping teachers to develop broader strategies for teaching mathematics. We have emphasized strict time allotments for each lesson component in order to have comparable procedures for evaluating the impact of the program on students. Although the general time allocations are practical (for example, the relative amounts of development and seatwork), we feel that the distribution of time and emphasis on particular lessons may vary greatly. That is, the framework is an average or generalized approach to teaching mathematics. It is now time to explore these instructional variations more fully, particularly in terms of the different types of mathematics content being presented.

The collective results of studies in our research program as well as those obtained elsewhere (Stallings, 1980; Anderson, Evertson and Brophy, 1979; Evertson, Anderson, Anderson, and Brophy, 1980; Program on Teacher Effectiveness, 1978) provide evidence that general training programs can have impact upon mean classroom performance. However, there are sufficient data to suggest that general treatment programs are apt to have different levels of impact on different types of teachers and students (Janicki and Peterson, 1981; Ebmeier and Good, 1979). Such results call for both a need to understand why programs affect different combinations of teachers and students in different ways and to develop procedures for developing more differentiated instructional programs.

We are pleased that our training program has had some success. However, it is important to reiterate that different teachers, various organizational structures, and diverse types of students have interacted in various ways with the program to affect the pattern of results. Much more information is needed about how these context conditions influence the program and the ways in which the general strategies and structures can be calibrated to fit into particular contexts.

Theory. Past research has shown that teachers vary in their behavior and in their effects on students. It is now time to synthesize the findings from our research program and research elsewhere, and to identify models for studying particular contexts. We must also learn how to adapt mathematics lessons to individual students and to particular types of content. However, as noted in Appendix 2, such a synthesis of empirical studies will be very complex. To guide the synthesis of present results, to direct future research, and to gain insights into mathematics teaching, we will need to develop new theoretical constructs.

Since the Missouri Mathematics Program focused on whole-class instruction, it is difficult to speculate about its effects on particular learners or for particular content. Nevertheless, it might be instructive to present some hypothetical comments about why the Missouri Program has appeared to work at a general level. These ideas have not yet been tested, but hopefully will be topics of future research.* The following comments are taken from Good (in progress).

> ...We have evidence that the Missouri Mathematics program in general had positive impact upon the mean performance of students in experimental classrooms, but we have no data to explain why the program worked. I suspect that the program had an impact because many elementary school teachers simply do not emphasize the meaning of the mathematical concepts they present to students, and they do not actively teach these concepts. Too much mathematics work in elementary schools involves some brief teacher presentation and a long period of seatwork. Such brief explanations for seatwork do not allow for meaningful and successful practice of concepts that have been taught, and the conditions necessary for students to discover or use principles on their own are also lacking.
>
> It seems plausible that the emphasis in our program upon the development stage of the lesson leads teachers to think more deeply about the concepts that they are presenting and to search more actively for better ways of presenting those concepts to students. Furthermore, given the way in which the development stage of the lesson is conducted, the program of instruction should allow teachers to see students' errors before they have a chance to practice those mistakes for a long period of time. This feature of the program seems to be especially desirable because some research has suggested that it is very difficult for students to tell teachers that they do not understand instruction. The clear development lesson would help students to understand more fully the concepts that they must master and how those concepts are related to other concepts that they have learned. The development phase of the lesson thus helps both teachers and students to develop better rationale for learning activities and to develop a sense of continuity.

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*There is some recent research evidence to illustrate that students are more attentive in treatment than control classes (Harre, 1980).

The controlled practice portion of the lesson aids both teachers and students in understanding whether the basic concepts and mechanics are being understood. Such information especially allows teachers to correct and to re-teach aspects of the lesson so that students develop appropriate conceptual understandings and skills prior to sustained practice. Also, it is hypothesized that students would be much more active thinkers during the development and controlled practice portions of the lesson. This is because students know that seatwork and their homework are intimately related to these activities. Hence, successful understanding during controlled practice leads to successful seatwork and successful homework. Checking of seatwork allows teachers one final opportunity to correct misunderstandings prior to the assignment of homework. Following successful practice, brief homework assignments should offer students positive learning experiences that both provide for better integration of material and also the development of more appropriate student attitudes about mathematics and their ability to learn it. In particular, students will probably conclude that increased personal effort during mathematics instruction leads to positive learning experiences. Students would thus be presenting more positive feedback to teachers about mathematics instruction (e.g., handing in completed homework and exhibiting positive verbal and non-verbal behaviors during mathematics instruction, which in turn increase teachers' expectations that they can present mathematics effectively, leading to renewed efforts on their part to carefully structure the mathematics lesson).

The preceding statements are only a few of the beliefs and hypotheses that we hold about why the mathematics program was working. It is important to note that these hypotheses need to be tested if we are to develop more adequate understanding of the antecedent conditions necessary for successful mathematics learning. For example, research is needed to determine if in fact experimental teachers identify more student errors and can more readily understand those mistakes during the development stage than do control teachers who use different teaching techniques. It would be equally important to determine whether students in experimental classrooms are more active thinkers during the development portion of the lesson than are students in control classrooms (perhaps by asking students to do problems immediately after the development portion of the lesson). Similarly, more research is needed concerning the conditions under which student errors are developmentally helpful and lead to increased student effort to integrate material, rather than debilitating and convincing students that they do not understand mathematics. When teaching effectiveness studies begin to examine their embedded assumptions by stating and testing the specific ways in which student learning is influenced, the conditions under which teaching and learning strategies are useful will become clearer than they are at present.

Clearly, these comments are meant to explain why the mathematics program was working in elementary schools. We need to consider the contextual differences between teaching in elementary schools and teaching in secondary schools, to include such contextual differentiations in our theoretical thinking, and to begin to test these theoretical notions. Investigators must consider the perspectives of teachers, students, and researchers as they formulate theories and design studies. Historically, research on teaching has tended to emphasize one set of variables at the expense of others. For example, sometimes detailed clinical interviews are conducted with students, but classroom observations of those students are not made, and teacher opinions about what was taking place during instruction are not measured. If we are to increase understandings of classroom learning, it will be necessary to incorporate the more immediate responses of teachers and students into the design of classroom research--one beginning effort is presently under way (Confrey and Good, in progress).

Integrating research paradigms. Although our research began with an attempt to identify what "effective" teachers did in the classroom, we do not believe that this is necessarily the best way to understand teaching. We felt at the time that little was known about what takes place during instruction in elementary school mathematics and we wanted to understand the phenomenon more fully.

We used a quantitative procedure for this purpose, and we think that such large-scale research may be useful for identifying interesting sources of naturalistic variance (e.g., teacher behavior, student behavior, student outcomes, etc.) which may be worth investigation. For example, mathematics educators are very interested in problem-solving behavior and "solutions" for improving instruction are frequently offered, yet we have no information about what teachers do when they are teaching problem solving.

We know from our own data that *some* teachers who group students for instruction in mathematics have very positive effects on students, as measured by standardized achievement tests. Because our research design focused upon the "extreme performers" in our sample (most of whom happened to be whole-class or large-group teachers), we cannot describe how more and less "effective" small-group teachers varied in their behavior. There are probably many other important questions that can be addressed effectively by quantitative/naturalistic procedures. However, experiences and outcomes that many would like to see occurring in schools, but which do not presently exist in natural practice, would require changing behavior, not merely studying it naturalistically (Good, 1980).

Quantitative strategies are useful for identifying potentially interesting classroom practices (although in some cases the "surface" exposed by such strategies may be misleading), and for describing such practices in rough outlines. However, quantitative strategies seem to be a poor methodology for explaining classroom patterns. Researchers interested in theory and in the design of instruction to fit a particular context will have to explain dynamic patterns of pupil and teacher responses and their mutual adaptation. Qualitative strategies are better for addressing such issues as social language in learning, the subjective interpretations of teachers and students, negotiated meaning, and reciprocal effects of classroom participants (see for example, Florio, 1979; Erickson, 1977).

Qualitative strategies offer no guarantee of increased understanding of classrooms. All too often in qualitative research investigators study only a few aspects of the environment in great detail while other variables potentially affecting classrooms are ignored. If new theories and more differentiated instructional programs are to emerge, more comprehensive and integrative strategies will have to be utilized. In new research efforts, quantitative strategies may be used to associate problems with an appropriate sample. For example, an investigator interested in the polarization of high- and low-achieving students in American classrooms might want to use quantitative strategies to determine whether there are classrooms where low-achieving students do not fall further behind high-achieving students as the year progresses. If such classrooms exist, they would provide an interesting contrast to classrooms in which increased polarization does occur. After potential samples are identified (and quantitative strategies would often be useful for this purpose), then quantitative and qualitative methodologies could be used in combination to explore the problem. We are clearly advocating that researchers match their research strategy to the problem being studied. We are also suggesting that research become more integrative. It seems unfortunate that research focuses upon social learning or academic learning; or upon students or teachers; or upon management or instruction.

A potentially useful approach at this time would be a more comprehensive study of the major aspects of classrooms simultaneously (teacher perception and behavior; student perception and behavior; curriculum content; social learning). As the scope and breadth of studies increase, it may also be advantageous to increase the range of competencies that individuals bring to the research task. Individual experts can work on various aspects of a large study. Obviously, there are limits to the number of variables which can be included in a single study and only certain aspects of classrooms can be examined, even with large amounts of time and money. There are also problems in securing and maintaining working relationships in cross-discipline research teams. However, despite these problems, it seems important for researchers to study more classroom variables than they have in the past. One means by which such expansion of research might take place is a cross-discipline research team whose individual members share a commitment to a common research problem, even though they have different methodological and substantive skills and insights.

Table 1

Summary of Key Instructional Behaviors*

Daily Review (First 8 minutes except Mondays)

a) review the concepts and skills associated with the homework

b) collect and deal with homework assignments

c) ask several mental computation exercises

Development (About 20 minutes)

a) briefly focus on prerequisite skills and concepts

b) focus on meaning and promoting student understanding by using lively explanations, demonstrations, process explanations, illustrations, etc.

c) assess student comprehension

  1) using process/product questions (active interaction)

  2) using controlled practice

d) repeat and elaborate on the meaning portion as necessary

Seatwork (About 15 minutes)

a) provide uninterrupted successful practice

b) momentum - keep the ball rolling - get everyone involved, then sustain involvement

c) alerting - let students know their work will be checked at end of period

d) accountability - check the students' work

Homework Assignment

a) assign on a regular basis at the end of each math class except Fridays

b) should involve about 15 minutes of work to be done at home

c) should include one or two review problems

Special Reviews

a) Weekly Review/Maintenance

  1) conduct during the first 20 minutes each Monday

  2) focus on skills and concepts covered during the previous week

b) Monthly Review/Maintenance

  1) conduct every fourth Monday

  2) focus on skills and concepts covered since the last monthly review.

*Teachers were also requested to slightly pick up their pace through the textbook material.

Table 2

Mean Percent of Occurrence of Selected Implementation Variables for Treatment and Control Group Teachers and the Correlation of These Variables with Teachers' Residualized Gain Scores on the SRA Mathematics Test, Field Experiment I

		Treatment $\bar{x}$	Control $\bar{x}$	p-Value	Correlation	p-Value
1.	Did the teacher conduct review?	91%	62%	.0097	.37	.04
2.	Did development take place within review?	51%	37%	.16	.10	.57
3.	Did the teacher check homework?	79%	20%	.0001	.54	.001
4.	Did the teacher work on mental computation?	69%	6%	.001	.48	.003
5.	Did the teacher summarize previous day's materials?	28%	25%	.69	.20	.23
6.	There was a slow transition from review.	7%	4%	.52	-.02	.91
7.	Did the teacher spend at least 5 minutes on development?	45%	51%	.52	-.08	.65
8.	Were the students held accountable for controlled practice during the development phase?	33%	20%	.20	.12	.50
9.	Did the teacher use demonstrations during presentation?	45%	46%	.87	-.15	.41
10.	Did the teacher conduct seatwork?	80%	56%	.004	.27	.13
11.	Did the teacher actively engage students in seatwork (first 1½ minutes)?	71%	43%	.0031	.32	.07
12.	Was the teacher available to provide immediate help to students during seatwork (next 5 minutes)?	68%	47%	.02	.28	.11
13.	Were students' held accountable for seatwork at the end of seatwork phase?	59%	31%	.01	.35	.05
14.	Did seatwork directions take longer than one minute?	18%	23%	.43	-.02	.92
15.	Did the teacher make homework assignments?	66%	13%	.001	.49	.0[illegible]

Table 3

Pre Project and Post Project Means and Standard Deviations for Experimental and Control Classes on the SRA Mathematics Achievement Test, Field Experiment I

I. All Treatment and All Control Teachers

	Pre Project Data			Post Project Data			Pre-Post Gain		
	Raw Score	Grade Equivalent	Percentile	Raw Score	Grade Equivalent	Percentile	Raw Score	Grade Equivalent	Percentil
Experimental									
Means	11.94	3.34	26.57	19.95	4.55	57.58	8.01	1.21	31.01
Standard Deviations	3.18	.51	13.30	4.66	.67	18.07			
Control									
Means	12.84	3.48	29.80	17.74	4.22	48.81	4.90	.74	19.01
Standard Deviations	3.12	.48	12.43	4.76	.68	17.45			

II. Control Whole Class Teachers and Control Group Teachers

	Pre Project Data			Post Project Data			Pre-Post Gain		
	Raw Score	Grade Equivalent	Percentile	Raw Score	Grade Equivalent	Percentile	Raw Score	Grade Equivalent	Percenti
Whole Class Control									
Means	11.70	3.30	25.30	16.20	3.98	43.00	4.50	.68	17.70
Standard Deviations	2.58	.40	10.15	4.96	.68	18.09			
Group Control									
Means	14.78	3.77	37.50	20.38	4.64	58.77	5.60	.87	21.27
Standard Deviations	3.14	.48	12.68	3.12	.47	11.56			

Note. SRA = Science Research Associates

Table 4

Means and Standard Deviations on Pre and Post SRA and Post Problem Solving Test by Instructional Group and by Classroom Organization, Field Experiment II

	Pre SRA		Post SRA		Pre-Post Change on SRA	Post Problem	
	$\overline{x}$	SD	$\overline{x}$	SD		$\overline{x}$	SD
Control	26.80	4.1	29.65	3.7	2.85	14.71	1.6
Semi	27.35	4.1	30.56	4.0	3.21	14.86	1.8
Open	25.36	2.7	27.70	2.6	2.34	14.55	.85
Special	27.26	5.9	29.78	4.3	2.52	14.56	2.1
Treatment	25.03	5.0	28.96	4.8	3.93	14.90	2.0
Semi	25.22	4.2	28.71	4.8	3.49	15.17	1.4
Open	20.41	1.3	26.01	4.9	5.60	13.13	3.6
Special	27.44	6.3	31.18	4.7	3.74	15.46	1.8

Table 5

Analysis of Variance Results on Adjusted Mean* Problem Solving Test Scores (Using Pre SRA Scores as a Covariate) Between Treatment and Control Classrooms With Open Classes Dropped, Field Experiment II

Source	DF	MS	F	Probability
Treatment Condition	1	5.45	6.77	.015
Error	25	.81		

* Note the adjusted mean for the control group was -.45 and for the treatment group .45.

Table 6

Teacher Sample: Junior High Mathematics Study

Partnership Teachers	
# Teachers	6
# Regular eighth-grade math	10
# Eighth-grade algebra	1
# Ninth-grade algebra	3
Treatment Group	
# Teachers	5
# Regular eighth-grade math	7
# Eighth-grade algebra	0
# Ninth-grade algebra	2
Control Group - Observation	
# Teachers	5
# Regular eighth-grade math	9
# Eighth-grade algebra	3
# Ninth-grade algebra	1
Control Group - No Observation	
# Teachers	3
# Regular eighth-grade math	5
# Eighth-grade algebra	0
# Ninth-grade algebra	0
Combined Partner and Treatment	
# Teachers	11
# Regular eighth-grade math	17
# Eighth-grade algebra	1
# Ninth-grade algebra	5
Combined Control - Observation and No Observation	
# Teachers	8
# Regular eighth-grade math	14
# Eighth-grade algebra	3
# Ninth-grade algebra	1

Table 7

Selected Implementation Scores by Junior High Teachers*

Teacher Number	Treatment or Control	Average Number of Minutes on Mental Comp.	Average Number of Minutes on Verbal Problem Solving	Average Number of Minutes on Development	Average Number of Minutes on Practice Seatwork	Development Overall Quality	Average Implementation Score
1	T	2.00	8.86	3.43	5.00	2.50	2.57
2	T	7.13	8.53	8.20	9.27	3.92	3.20
3	T	0.00	0.47	6.07	18.20	2.25	1.20
4	T	3.73	6.47	7.80	9.20	3.82	3.47
5	T	4.94	2.75	13.25	4.69	3.14	2.38
6	T	0.00	2.79	4.43	12.86	2.75	2.21
7	T	1.13	4.53	14.53	16.20	3.23	2.33
8	T	1.25	1.75	18.19	11.00	3.85	2.44
9	T	2.00	2.47	6.40	13.93	3.63	3.07
10	T	0.00	4.67	11.80	11.33	2.13	1.89
11	T	2.00	3.00	11.21	12.14	3.17	1.93
12	C	0.57	3.21	14.00	11.36	3.54	1.93
13	C	0.00	4.57	9.71	16.14	4.57	1.86
14	C	0.00	0.00	5.29	22.50	2.44	1.21
15	C	0.00	0.40	13.80	14.53	2.50	1.80
16	C	0.00	0.00	17.00	11.00	2.57	2.33
17	C	0.00	0.00	6.75	26.81	2.15	1.06
18	C	0.00	1.86	1.71	8.71	3.33	1.43
19	C	0.00	0.77	15.46	18.54	3.73	2.77

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* Teachers 1-6 are partnership teachers; 7-11 are treatment teachers; 12-16 are regular control teachers; 17-19 are control teachers who did not attend the orientation meeting.

Table 8

The Correlation and p-values of Average Implementation, Time on Mental Computation, and Time on Verbal Problem Solving with Residual Gains in Computation, Problem Solving, and Attitudes Toward Mathematics

	Average Implementation	Average Instructional Minutes Spent on Mental Computations	Average Instructional Minutes Spent on Verbal Problem Solving
Computational Residual (N = 19)	.16 (NS)	.24 (NS)	.45 (.05)
Problem-Solving Residual (N = 19)	.26 (NS)	.49 (.05)	.51 (.02)
Attitude Residual (N = 16)	.56 (.02)	.34 (NS)	.43 (.09)
Average Implementation	- - -	.63 (.003)	.58 (.008)
Time on Mental Computation	.63 (.003)	- - -	.63 (.003)
Time on Problem Solving	.58 (.008)	.63 (.003)	- - -

Table 9

Pre, Raw, and Adjusted Means for Junior High Treatment and Control Classes
On Pre-Math Test and on Post-SRA Subtest Scores

	N	Pre-achievement	Post $\bar{x}$ Computation	Adjusted Computation	Post $\bar{x}$ Problem Solving	Adjusted $\bar{x}$ Problem Solving
Treatment	21	47.65	29.75	29.84	21.90	21.98
Control	18	48.37	28.97	28.86	20.99	20.83

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Appendix 1

The Missouri Mathematics Treatment Program

Teachers Manual:

Missouri Mathematics Effectiveness Project

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I. Introduction

We believe it is possible to improve student performance in mathematics in important ways. We look forward to your help and co-operation in implementing the program that we have discussed at the workshop and which is outlined again in the material that follows. Through your efforts we believe a significant difference in student performance will be made.

We do not believe that any single teacher behavior will make a critical difference in student learning, but we do feel that several behaviors in combination can make a major impact. In the material that follows, we present a system of instruction that, if followed daily, will enhance student learning.

In general, we feel that the plan should be followed each day. However, we also realize that special circumstances will force you to modify the plan on occasion. Still, it is important that you follow the daily plan as frequently as you can.

For purposes of clarity, we will discuss each part of the teaching program separately. However, once again we want to emphasize that the program works when all parts are present. To maximize your opportunity for obtaining a clear picture of the instructional program, the program is summarized in Table 1. The rationale for each part and how the pieces fit together will be discussed at a later point in the handbook.

Table 1
Summary of Key Instructional Behaviors*

Daily Review (First 8 minutes except Mondays)

- a) review the concepts and skills associated with the homework
- b) collect and deal with homework assignments
- c) ask several mental computation exercises

Development (About 20 minutes)

- a) briefly focus on prerequisite skills and concepts
- b) focus on meaning and promoting student understanding by using lively explanations, demonstrations, process explanations, illustrations, etc.
- c) assess student comprehension
  - 1) using process/product questions (active interaction)
  - 2) using controlled practice
- d) repeat and elaborate on the meaning portion as necessary

Seatwork (About 15 minutes)

- a) provide uninterrupted successful practice
- b) momentum - keep the ball rolling - get everyone involved, then sustain involvement
- c) alerting - let students know their work will be checked at end of period
- d) accountability - check the students' work

Homework Assignment

- a) assign on a regular basis at the end of each math class except Fridays
- b) should involve about 15 minutes of work to be done at home
- c) should include one or two review problems

Special Reviews

- a) Weekly Review/Maintenance
  - 1) conduct during the first 20 minutes each Monday
  - 2) focus on skills and concepts covered during the previous week
- b) Monthly Review/Maintenance
  - 1) conduct every fourth Monday
  - 2) focus on skills and concepts covered since the last monthly review

* Definitions of all terms and detailed descriptions of teaching requests will follow.

II. Development

Variable Description

The developmental portion of the mathematics period is that part of the lesson devoted to establishing comprehension of skills and concepts. It should be viewed as a continuum which runs from developing understanding to allowing for meaningful practice in a controlled setting. During all stages of the developmental portion of the lesson, both ends of the continuum may be present to some degree. However, in general, the comprehension emphasis with very little practice will come at the initial part of the lesson, then toward the middle of the lesson, practice with process feedback from the teacher will become quite prominent, and finally in the latter portion of the lesson there will be controlled practice with meaningful explanations given as necessary.

The role of the teacher in the first part of the lesson, the comprehension phase, is to use instructional strategies that help students understand clearly the material being studied that day. In this portion of the lesson emphasis is placed upon comprehension rather than rote memorization. Activities which often focus on comprehension include teacher explanations and demonstrations, and they may include use of manipulative materials to demonstrate processes and ideas, use of concrete examples in order to abstract common features, making comparisons and searching for patterns, and class discussions.

During the middle portion of the lesson, the number of questions posed to students may increase as the teacher begins to assess comprehension and provides them an opportunity to model processes already demonstrated and to verbalize the understanding they have developed. During this phase of the lesson, the teacher may decide that further explanations and demonstrations are necessary or decide that controlled practice is appropriate since students seem to understand what they are doing.

In the controlled practice phase of the lesson the emphasis is on increasing proficiency; that is, increasing speed and accuracy. However, meaningful feedback is still given as necessary or requested.

Problem

Many problems arise in math classes in which teachers give too little attention to development. Students exposed to such teaching frequently attempt to memorize rules for doing things and concentrate on mechanical skills. These rules have no meaning for the student (because developmental work was not done) and, thus, they are easily forgotten especially when new sets of rules are "learned." When students do not understand what they are doing, each new problem causes them great difficulty. Often the comment, "We haven't done any of these before" is heard. When students learn without understanding, the ability to transfer skills to new situations is greatly reduced. Other negative results such as the inability to detect absurd answers and loss of self-confidence also occur. Thus, there are many compelling reasons to include a large measure of developmental work in mathematics lessons.

Teaching Practice

Initially, the teacher should focus briefly upon prerequisite skills that students may need for the lesson. Then the major aspect of the meaning portion of the development lesson occurs: active demonstration of the concept, idea and/or skill that is being focused upon in the lesson, etc. Teachers need to demonstrate actively the process, so that students can comprehend the learning goal. You need to be cautious about moving too quickly into the assignment of problems and practice without providing students with an adequate conceptual orientation.

After the active demonstration and explanation by the teacher (and we recommend that 10 minutes minimum be spent on this), the teacher should begin to assess student comprehension. There are two primary ways to do this. First, teachers may ask oral questions. In general, we recommend that teachers generally ask brief product oriented questions. Product questions are questions that assess whether or not the student can produce the correct answer (see appendix A for a complete description). Teachers can maintain an emphasis upon meaning by frequently providing process explanations themselves after students respond ("Yes, Tina, that's right because . . .").

The second way that teachers can assess student comprehension is by having students work practice problems. However, it is important to recognize that the role of a practice problem in this stage of the lesson is not to build up student speed and accuracy per se, but rather, the goal is to allow teachers to assess student comprehension. Hence, the assignment of problems in this stage should be limited to a single, brief problem followed by teacher assessment and explanation and then the provision of another brief problem assignment. In general, this stage of the lesson can be completed in 3-5 minutes.

If your questions or assigned problems reflect a moderate degree of student difficulty, then you should repeat the meaning portion of the lesson. If possible, use different examples; however, if this is not possible, verbatim repetition of the initial portion of the lesson is better than to proceed to controlled practice and seatwork when students are confused. Such a situation guarantees that students will practice errors.

If assessment of student comprehension is largely satisfactory, *then* teachers should proceed to the controlled practice portion of the development lesson. Now, the teacher provides opportunity for students to develop increased speed, accuracy, and proficiency in completing problems of a specific type. However, the practice is still heavily controlled (unlike seatwork practice which will be discussed in the following section).

During controlled practice, teachers should assign only one or two problems at a time. *Students should not be asked to work longer than a minute without feedback about the correctness of their responses.* The reason for this is that during the controlled part of the lesson the teacher is still trying to identify and correct any student misunderstanding. Too often many students are left to watch while a few students demonstrate a problem on the board. A great deal of practice time is lost this way and often the involvement of some students in the lesson (momentum) is lost as they become engaged in side conversations and distractions.

During controlled practice exercises, teacher accountability and alerting should be *immediate* and *continuous*. By alerting, we mean teacher behaviors that remind students that they should be doing work and that it will be checked. For example, if the teacher sends 3-4 students to the board to demonstrate the problem that students have just worked at their desks, the teacher might say, "Now the rest of you do these two new problems at your desk and I'll check them in a minute." Such teacher behavior maintains student momentum. Instead of watching classmates write on the board, they have their own work to do and they are alerted to the fact that they will be held responsible for the work.

By accountability (more on this when we describe the seatwork portion of the lesson) we mean the actual checking of student responses. For example, while students put their work on the board, the teacher could look at the work of students who remain at their desks and check the problems that they were to have completed. Furthermore, the teacher can call on students to provide answers to practice problems, etc. Through such procedures, the teacher is able to assess when students are prepared to move to the seatwork portion of the lesson where they have a longer block of time for uninterrupted practice. A final important characteristic of the controlled seatwork portion of the lesson is that the practice is done in the context of meaning (e.g., the teacher is frequently providing process explanations "Yes, that's right because . . ."). Although the teacher is beginning to work for speed and accuracy, some attention is still being paid to students' understanding of the concepts, ideas, and skills that are being developed.

In summary, the development part of the lesson calls for the following teacher behavior:

(1) Review briefly and/or identify prerequisite skills.

(2) Focus upon the development of meaning and comprehension using active demonstration and teacher explanation.

(3) Assess student comprehension (ask questions/work on supervised practice).

(4) Repeat meaning portion of the lesson as necessary (using different examples and explanations if possible).

(5) Provide practice opportunities for students.

  (a) Practice should be short (one or two problems at a time).

  (b) Students should be held responsible for assigned practice problems.

  (c) Practice should be performed in a meaningful context (teacher provides frequent process explanations).

  (d) When success rate is high, move students into seatwork portion of the lesson where students have an opportunity for uninterrupted practice.

XII. Seatwork

Variable Description

Seatwork refers to practice work that students complete individually at their desks. Since seatwork practice follows the controlled practice part of the development lesson, students should know the purpose of assigned problems and how to do them when they begin to work. The role of seatwork practice is both important and easy to describe. Seatwork assignments allow students to practice, on their own, problems and principles that you have just actively taught. Seatwork provides students with an opportunity for immediate and successful practice. This practice experience allows students to achieve increased proficiency and to consolidate learning. New material or review work should not be assigned during the seatwork portion of the lesson.

Problem

Often a great deal of time is wasted when students work on problems individually. Indeed, research has consistently shown that students show less involvement (amount of time that students actually spend working on problems) during the seatwork portion of the lesson than during the active teaching portion of the lesson. Too often teachers stop active supervision after they make the seatwork assignment. Two of the more common ways that teachers stop supervision are by doing desk work, grading or by providing extended feedback to a single student (before all students are working on the task). Such behavior virtually guarantees that teachers cannot provide the type of supervision that students need if they are to begin to work productively. The first teaching task is to get students started on the seatwork. Often students do not use seatwork time productively simply because the teacher does not obtain their attention initially.

In addition to the problem of not "demanding" students to start work, some teachers create a problem by moving from the development portion of the lesson to seatwork with such abruptness that it is not surprising that students do not begin to work immediately (e.g., four students spring to the pencil sharpener, two students search for materials, and three students begin a private conversation). Momentum needs to be maintained throughout all stages of the lesson. When momentum is lost, students are apt to take a psychological break and once momentum (student attention and involvement) is lost, it is difficult to "recapture." Teachers who end the development portion of the lesson with a controlled practice segment have done much to ease the transition from the group lesson to individual seatwork.

Teaching Request

Given that the role of seatwork is to provide opportunity for successful practice, we recommend that about 10-15 minutes each day be allotted for seatwork. Ten to fifteen minutes allows sufficient time for students to work enough problems to achieve increased proficiency but not so long as to bring about boredom, lack of task involvement, and the behavioral problems that soon follow when students are bored or frustrated. Frustration should be minimal in seatwork activity because the problems students are asked to do are a direct extension of the development part of the lesson. If practice time does not exceed 15 minutes, few students are likely to be bored.

The number of problems assigned should take most students only 15 minutes to complete. Hence, approximately 75 percent of your students should be able to complete the work within the allotted 15 minutes. In making the seatwork assignment, emphasis should be placed upon accurately working as many problems as possible within the allotted time. Low achievers who remain on task and do accurate work have done well and should know that they have done well. That is, the criterion to communicate to students is to keep working and to do as many problems accurately as they can.

To help optimize the effectiveness of seatwork, three general principles should be observed. The first principle, momentum, has already been discussed indirectly. By momentum we mean keeping the ball rolling without any sharp break in teaching activity and in student involvement. Teachers can achieve momentum by ending the development portion of the lesson by working problems similar to the ones that students are asked to work individually and by starting students on individual work with a simple and direct statement. "We've worked problems 1 and 3. Now, individually, at your desk do problems 5-15. Work as many problems as you can, and we'll check our work in 15 minutes. Remember doing the problem correctly is more important than speed." Following such a statement, you should actively monitor all students. Before providing feedback to individual students, make sure all students are engaged in the seatwork.

If some students do not begin working immediately, walk to their desks and if your physical presence doesn't initiate student work as it usually will, then quietly say something like "Frank, it's time to do the problems." After all students are working on the problems (the ball is rolling), you can then attend to the needs of individual students. In general, students should get immediate feedback and help when it is needed. Thus, it is usually reasonable to allow students to approach you when they have a question or problem. However, when presenting feedback to individual students, keep in mind the general principle of momentum. You have to provide feedback and conditions that allow most students to stay on task (keep working). Hence, it is not advisable to continue to provide lengthy feedback to an individual while several students are waiting for teacher feedback before they can continue to work.

Alerting is a second principle to observe during seatwork. Alerting behaviors tell students that they will be held accountable for their work. Often students engage in off task behavior because they are not alerted to the fact that they will have their work checked at a specific point in time. If students are assigned seatwork that won't be checked until the following day (or when it is not checked at all), students are not likely to be highly engaged in seatwork. A statement like, "We'll check the work at the end of the period.' alerts students to the fact that there is reason to engage in productive work immediately. A statement at the beginning of the seatwork is sufficient. Repeated statements are apt to interfer with students' work concentration. Public announcements should not occur during seatwork. Once you have students working it doesn't make sense to distract them.

Accountability is the third principle to observe during seatwork. Alerting, as we noted, is a signal to students that their work will be checked. Accountability is the actual checking of the work. It is important that your accountability efforts do not interrupt the seatwork behavior of students. During the controlled practice part of the lesson (see development section), accountability is immediate. However, during the seatwork portion of the lesson, students are to be working more independently and those students capable of doing the work need time for uninterrupted practice. Public accountability needs to be delayed until the end of the lesson. A teacher's public questions during this stage of the lesson are very disruptive. For example, when the teacher asks a public question (e.g., "How many of you have done the first four problems?," "What's the answer to the second problem?," etc.) all students stop work and once momentum is lost, some students will take much time before resuming their work. Furthermore, questions like "How many of you have finished the first four?" may make students anxious and distract them from task behavior if they have not worked the first four problems. Occasionally, you may need to stop seatwork practice to correct a common misunderstanding. In general, these errors should be corrected during the development (controlled practice) phase of the lesson. Public statements (except for necessary behavioral management) should be avoided. If most students are not ready for seatwork practice, then you should stay in the controlled practice part of the lesson. Such behavior will help students develop the following attitude toward seatwork: "I can do the problems and now it is time for me to apply myself."

Perhaps the most direct and easiest way to hold students publicly accountable without disrupting seatwork is to call on individual students at the end of the lesson. Checking students' work at the end of the period also provides the teacher with a chance to spot any systematic mistakes that students are making and to correct those misunderstandings. Hence, when your students are assigned their homework, conditions should be set so that the homework provides for additional and relatively successful practice.

Specifically, we are asking you to get student involvement immediately after making a seatwork assignment. Continue to monitor and supervise all students until they are engaged in assigned work (the first minute or two). Early in the seatwork period (the first three to five minutes), be available for students when they need feedback. Toward the end of the seatwork period, try to get to the desks of some low achievers to see if they are making any systematic errors and to provide feedback as necessary. At the very end of the seatwork period, hold students accountable for their work by asking individual students to give the answer to a few of the assigned problems. This checking of answers should be very rapid and you need only check 3 or 4 of the problems (check one or two problems at the first, in the middle and at the end of the assigned work). If misunderstandings are corrected here, the homework should be a successful practice experience for most students.

When conducting the review of seatwork, it is generally advisable to call on low achievement students to provide answers only to the first few problems assigned so as not to frustrate them for failure to complete all problems, but be sure to increase seatwork expectations for these students as the year progresses.

Finally, all seatwork should be collected. This helps encourage students to work productively because they know that they are held accountable for the work assigned during seatwork. Because of the way teachers have used seatwork in the past, many students have built up the expectation that seatwork is a time to relax and waste time. Taking up the seatwork will help students to adjust to the expectation that seatwork is a time to apply themselves and to see if they can do the type of problems which will be assigned as homework. Although there is no compelling reason to grade seatwork, it is important to examine the papers to see if students are using seatwork time appropriately. If a student's work is unduly incomplete, impossible to read, etc., it would be important to mention this to the student so that he or she knows that you care about his seatwork performance.

After the seatwork is collected, the homework assignment is made. Delaying the assignment of homework helps to insure that students will do the work at a later point in time, hence, building distributed (repeated) practice into the mathematics programs. Research has consistently shown the superiority of distributed practice over mass practice in helping students to master and retain new concepts and skills.

IV. Homework

Variable Description

Mathematics homework is written work done by students outside the mathematics class period. It is usually done at home; thus, it is distinctly different from seatwork which is done within mathematics class time.

Problem

The emphasis on homework in schools over the years has varied considerably. Unfortunately, homework has been misused frequently. Sometimes the assignments were so long that students became bored and careless when working the assigned problems. No doubt some students' dislike for mathematics is in part associated with these lengthy assignments. The instructional value of long homework assignments is very questionable. If students make errors on the first few problems of the assignment, then by the end of the assignment they may have become more proficient in making those errors!

Other situations in which homework has not been used to its full potential are plentiful. In some schools homework is never given or so few problems are assigned that an excellent opportunity for distributed practice is wasted. Another undesirable situation occurs when homework is given primarily to please parents but without much attention to selecting problems and assignments that will foster progress toward important objectives. But perhaps the most devastating misuse of homework is when children are assigned problems for which inadequate background has been developed in class. While long assignments often lead to frustration, this latter situation always leads to frustration and negative attitudes toward the mathematics class.

Another situation which detracts from the value of homework assignments happens when the teacher fails to stress the importance and value of the problems assigned. This can be done directly by not commenting on the importance of assignments or indirectly by not scoring or collecting assignments.

If spite of these misuses of homework, homework can be an important part of mathematics learning if certain guidelines are followed. Research suggests that giving homework to students on a regular basis may increase achievement and improve attitudes toward mathematics. Short assignments have been found to be most effective and some variety in the type of homework is helpful. Also, if a teacher gives importance to the homework through oral comments and by scoring papers regularly, then students frequently respond by completing their assignments with greater care.

Teaching Request

Because of the important role that homework can play in improving student performance in mathematics, we would like to have you do the following during the study:

1. At the very end of the math class period on Monday through Thursday, give a homework assignment which is due at the beginning of the class period the following day.
2. Each assignment should require about 15 minutes of outside class time. Within this time frame, assignments will probably average about eight problems per day depending on the kinds of problems being assigned. A typical assignment is shown in Appendix B.
3. The primary focus for an assignment should be on the major ideas discussed in class that day. Also each assignment given on Tuesday and Wednesday should include one or two review problems from the current week's work.

4. Each assignment given on Thursday should be primarily devoted to review problems from the current week's work. In order for sufficient practice to be given on the material discussed on Thursday, this assignment will be a bit longer than assignments for other days and will probably take about 20 minutes for most students to complete.

5. Typically, each assignment should be scored (number correct) by another student. Papers should then be returned to their owners for brief examination. Finally, papers should be passed forward so that the scores can be recorded in the grade book.

6. The assignments given should be recorded daily in the Teacher's Log.

The short homework assignments complement seatwork by distributing practice over time without putting undue time pressure on students. Short assignments help hold student interest; adding variety to assignments is also helpful. This can be done by embedding the problems to be worked in different formats such as games, puzzles, codes, and so on. Appendix C illustrates this idea. Another component of variety might be to have students check their work. Multiplication problems can be checked by doing division, addition problems by doing subtraction, and so on. Variety can also be introduced by giving differentiated assignments. For example, some students could be given ten easy problems, while other students are given six problems of a more difficult nature.

The scoring and recording of grades on all homework assignments are designed to emphasize the importance of homework and to provide regular feedback to students and teachers regarding progress being made by each student. It is important to realize that there are a number of efficient ways to score homework other than the teacher's going through the papers individually. For instance, students can exchange papers or score their own papers. Either of these procedures is improved if students are expected to have their homework completed and ready to be scored at the very beginning of math time. Efficiency is also improved if answers are prepared in advance by the teacher in written form (transparency, blackboard) and then shown to the students. Otherwise, the teacher may need to orally repeat each answer a large number of times.

Explanations and reteaching the homework must be somewhat limited if adequate time for discussion and practice of new material is to be available. This should not cause too much difficulty because most student difficulties and errors should have been remediated prior to the seatwork of the previous day.

A good strategy may be to quickly have children exchange and score papers, then have children indicate by raising their hands--how many missed problems #1, #2, and so on. Then you can rapidly work the one or two problems that caused students the most difficulty. Since there are usually only a small number of homework problems to be checked and discussed, this part of the lesson should be easily completed in two minutes. Finally, note that any reteaching that is not completed can be done during the weekly review that is discussed in the next section.

In the rare event that the checking of homework reveals numerous student errors, you should reteach the previous day's lesson beginning with development, then controlled practice, then seatwork, and finally a homework assignment on the same material. Under these circumstances you should not try to cover new material due to the very limited amount of time available to develop the new ideas.

You are requested to personally score the homework that is assigned on Thursdays. There are two reasons for this. First, the information gathered from this homework is to be used to structure the weekly review each Monday. Second, the focus of student scoring is of necessity on answers rather than kinds of errors being made. It is very important, however, that regular attention be given to the procedures and processes that students are using. This is especially true when they are making errors!

In connection with the scoring of Thursday's work, each student's paper should be analyzed for systematic error patterns. Systematic error patterns refer to incorrect procedures which are consistently used on a wide range of problems. In two-digit multiplication problems, for example, a student might consistently forget to "carry" the tens digit from the initial multiplication of the units digits. According to recent,research such errors are much more common than was once realized and,thus,spending time examining homework with them in mind can be very helpful in remediating some students' difficulties with mathematics. Further examples of common computational error patterns can be found in Appendix D. Since the particular errors you find probably will not be associated with groups of students, the remediation of such errors is usually best done on a one-to-one basis.

Homework is an important component of this program and since both students and teachers devote a considerable amount of time to it, it is recommended that homework count at least 25 percent of each student's math grade and that this information be communicated to them.

Parents are interested and should be informed about what is happening in school. Therefore, it is recommended that an explanation of the homework policy to be followed during the study be sent home to parents. A letter which could be duplicated and used for this purpose can be found in Appendix E.

Homework is explicitly related to each of the other components of the study in a number of ways. With an increase in development time, it provides an opportunity to supplement the practice part of the lesson. It is structured such that practice is distributed over time and students have an opportunity to correct difficulties encountered in seatwork. The homework provides important information for structuring the specific details to be covered in the review component. It is also related to the pacing variable in that it allows some necessary work to be done outside of the time regularly scheduled for math.

V. Special Review/Maintenance*

Variable Description

Children forget. It is imperative, therefore, that ideas be reviewed and skills maintained on a systematic basis in elementary school mathematics. Reviewing ideas may involve the teacher stating and explaining properties, definitions, and generalizations and the students recalling the appropriate term or name. These roles occasionally may be reversed (where the teacher supplies a term and the students illustrate and explain), but the focus should generally be developmental in nature. That is, there should be a strong emphasis on meaning and comprehension. Similarly, skills need to be practiced with regularity in order that a high level of proficiency be maintained. The focus should be developmental in nature; comprehension again is an important component.

Problem

When discussing children's performance in mathematics, frequently the comment is made that many have not mastered the basic skills. From this it is concluded that teachers do not spend enough time teaching basic computation. But this conclusion often is not valid because the inability to perform may not be associated with the initial learning but rather with a lack of maintenance. Newly learned material is particularly susceptible to being forgotten, but even material thought to be "mastered" is sometimes lost. For example, many fourth grade teachers have had the experience in which a student seems to have mastered his basic multiplication facts, indeed, he or she can recall them with almost 100 percent accuracy but four weeks later seems to have forgotten a great number of them.

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*The review discussed here is in addition to the brief (1-4 minute) daily review that we will discuss later in the handbook.

Teaching Request

To minimize this problem and similar problems, we are asking that you incorporate review/maintenance sessions regularly into your mathematics instruction. Regularly in the sense that each Monday you have a Weekly Review/maintenance session and every fourth Monday you have a Cumulative Review/maintenance session. The purpose of the two types of review sessions is to help students retain concepts and insights.

Weekly Review/maintenance. The following things are necessary to do if the review/maintenance component is to be implemented effectively:

1. The first one-half of each Monday's math period (roughly 25 minutes) should be devoted to review/maintenance.
2. The focus should be on the important skills and concepts covered in math during the previous week. The suggested order for covering these skills and ideas are:
    a. Those that are thought to be mastered and can be done very quickly.
    b. Those that need additional development and practice as identified from the analysis of the Thursday homework assignment.
    c. Those that need additional work (as identified during this review session).

Most of the important skills and concepts that should be reviewed can easily be identified by examining the homework assignments from the previous week. That is, these homework assignments deal with each important data or skill; thus, reviewing them will assist you in identifying important topics. It is of utmost importance that all major ideas covered during the week be reviewed. Reviewing ideas that students have "mastered" the previous week helps to guarantee that ideas will be retained. Areas in which some reteaching is definitely needed should be identified in advance by the teacher from an analysis of the Thursday homework assignment and handled during the second portion of this designated review segment.

There are many ways that this maintenance program can be successfully organized. One important attribute of any effective organizational scheme is active student involvement. In most teaching situations, it is important to avoid situations that involve only one student in checking problems because such a procedure is usually ineffective and boring to most children. This is especially true in a review situation in which students are already familiar with the problem. One scheme that we highly recommend (because it overcomes this difficulty) is one in which the teacher presents an idea or problem and then allows students to work individually at their desks until most arrive at an answer. Finally, answers are checked (children are held accountable), and someone explains or demonstrates how to arrive at the answer (in many cases by using the chalkboard at the front of the room).

Cumulative Review/maintenance. This aspect of the review/maintenance program can best be implemented in the following way:

1. Every fourth Monday the entire math period should be devoted to a cumulative review/maintenance session.
2. This review should encompass the work of the previous four weeks and thus replace the regular Monday maintenance/review session.

This session provides an opportunity to reteach ideas that have given difficulty over the past four weeks. It will be particularly useful to those students who have difficulty acquiring skills and ideas on initial exposure.

The interest in and value of this review session can be greatly enhanced by structuring it in an interesting format such as a game, contest, or quiz show.

Postscript

On occasion, it may be desirable to reschedule a review for a day other than Monday. For example, if by not reviewing on a Monday you can complete a chapter or unit, by all means do this and simply conduct your review on Tuesday. If it becomes necessary for you to reschedule a review, please make a note of it in the log so that we are aware of it.

VI. Mental Computation

Variable

Mental computation is computation that is done without the aid of pencil and paper (or minicalculator). The process is done by the most powerful computer of all, the human brain. Mental processing is often vastly different than pencil and paper calculation. For example, in pencil and paper addition situations the calculation always goes from right to left. The student asked to solve 41 + 12 on paper is going to move mechanically from right to left. However, in a mental activity (the teacher says what is 41 + 12) the student may frequently move from left to right. First, the student does something to the tens column, then to the ones column, and then combines. We feel that the inclusion of some time for mental computation each day will help students to further develop their quantitative sense, to become more flexible in thinking and in approaching problem-solving situations. Furthermore such activities help students to detect absurd answers (e.g. when checking their written computation) and make estimations that are frequently needed in daily activities.

Problem

The attention given to mental computation and mental problem-solving has largely disappeared from the modern mathematics curriculum. At one point in time much emphasis was given to mental problem solving. This de-emphasis has occurred despite some research evidence which suggests that mental practice on a regular basis appears to be related to large increases in student achievement. If students are not given some work in mental computation, then they are missing a very important way to check their work (other than the time consuming and inefficient process of completely redoing the work).

Teaching Request

We would like for you to include 3-5 minutes on mental computation activities each day at the beginning of the lesson; the predevelopment part of the lesson will be described later in the handbook. Ideally, the material presented for mental resolution would be related to the content of the material being studied. During the study of subtraction, mental computation activities should focus on subtraction. However, some units that you study in the year will not lend themselves to this form of mental processing. During such a unit (e.g. geometry) it would be useful to rotate on a daily basis with the following types of mental computation activities: addition, subtraction, multiplication, division, and verbal problems.

The following examples will give you some ideas about the kinds of problems you may present to your students. Some of the examples here may be too easy or too difficult for your students. You should try to use problems which are challenging yet accessible to most students. It is a good idea to discuss how a problem might be solved mentally before students are asked to give solutions.

For example, for a problem like 6 x 12 you might suggest thinking as follows: "6 times 12, that's 6 times 10 plus 6 x 2, that's 60 + 12, 72." Then begin giving students problems one at a time to solve like 8 x 12, 6 x 15, and so on. It is worthwhile to mention to the students that there are many ways to solve problems mentally and the way you showed is but one way. Children should be encouraged to discuss their mental computation procedures.

Further illustrations of the kinds of problems which are appropriate are given below. You should generate other types of mental computation exercises for your students as well.

Addition

(1) 75 + 77 = _____

Think: 77 = 70 + 7. First add 70 to 75 (145) then add 7 to that sum (152).

or: Rename 77 as 70 + 7 and 75 as 70 + 5. Add the tens (140), add the ones (12), then find the total of the sums (152).

(2) 97 + 8 = _____

Think: How much do I add to 97 to get 100? The answer is 3. Since 8 = 3 + 5, first I add 3 to 97, and then add 5 to the sum.

(3) 243 + 104 = _____

Think: 104 = 100 + 4. First add 100 to 243 and then add 4 to the sum.

(4) 125 + 49 = _____

Think: 49 is 1 less than 50. Since 125 + 50 = 175, 125 + 49 = 174.

Subtraction

(1) 125 - 61 = _____

Think: 61 = 60 + 1. First subtract 60 from 125, and then subtract 1 from the difference.

(2) 105 - 8 = _____

Think: First subtract enough from 105 to get 100: 105 - 5 = 100 Since 8 = 5 + 3, subtract 3 more: 100 - 3 = 97.

(3) 425 - 97 = _____

Think: 97 = 100 - 3. First subtract 100 from 425, and then add 3 to the difference. 425 subtract 100 is 325, add 3 is 328.

Multiplication

(1) 20 x 36 = _____

Think: 20 = 2 x 10. Ten times 36 is 360, and 2 x 360 = 720.

or: 2 x 36 = 72, so 20 x 36 = 720

or: 20 x 36 that's the same as (½ x 20) x (2 x 36), or 10 x 72 = 720.

(2) 4 x 17 x 25 = _____

Think: Since the product of 4 and 25 is 00, these numbers are multiplied first. Then 100 is multiplied times 17.

(3) 32 x 50 = _____

Think: The product is unchanged if I double one factor and half the other factor. Thus, 32 x 50 is the same as 64 x 25 or 1,600.

(4) 4 x 53 = _____

Think: 53 = 50 + 3. Four times 50 is 200. Four times 3 is 12. So to find 4 x 53 add 200 + 12.

Division

(1) 84 ÷ 4 = _____

Think 84 = 80 + 4. 80 divided by 4 is 20 and 4 ÷ 4 is 1, so 84 ÷ 4 is 20 + 1 or 21.

(2) 396 ÷ 4 = _____

Think: 396 = 400 - 4. Since 400 ÷ 4 = 100 and 4 ÷ 4 is 1, the quotient is 100 - 1 or 99.

(3) 250 ÷ 50 = _____

Think: 250 ÷ 50 is the same as 500 ÷ 100 which is 5.

Verbal Problems

(1) Mr. Thomas has a debt of $120. If he pays $70 of it, how large a debt will he have left?

Think: I need to find 120 - 70 = _____
12 - 7 = 5, so 120 - 70 = 50.
50 is the answer.

VII. Instructional Pace

Variable Description

Instructional pace refers to rate. It may be thought of in terms of how quickly a class is moved through a given curriculum or in terms of how rapidly students are presented with particular topics. The pace associated with different teachers varies. Some teachers move through the curriculum faster than others.

Problem

Instructional pace may inhibit learning in several ways. At one extreme is the situation in which a teacher moves through the curriculum too quickly for learning to take place. At the other extreme is the teacher who plods along so slowly that many of the students are bored. Furthermore, some teachers, because of their slow pace, find themselves forced to cover so much material at the end of the year that they do not have time to build in the distributed practice which is essential if students are to retain the material. Research suggests that for most teachers efficiency could be improved if they increased their pace slightly. That is, there seems to be more of a tendency to procrastinate than to move forward. If the suggestions presented earlier in the manual are implemented in your teaching program, the important element of review and distributed practice should be fulfilled and you will probably be able to pick up the pace.

Teaching Request

For this variable we ask that you carefully consider your teaching behavior with respect to the instructional pace you set. Many of you will find that you can increase the pace somewhat and we ask you to attempt to do so.

The instructional strategies suggested in this study are such that if you speed up a bit too much, then you can resolve problems that arise through your regularly scheduled review/maintenance sessions.

VIII. Starting and Ending the Lesson

We have now discussed the major parts of the mathematics instructional program. Two aspects that we have not discussed explicitly are the start and end of the lesson.

The beginning portion of the lesson (Predevelopment) will have three parts: (1) a brief review, (2) the checking of homework, and (3) some mental computation exercises. We ask that all three of these activities be done within the first eight minutes of the class period. This may be difficult for teachers who slowly ease into the lesson, but it has been commonly observed that time is frequently used inefficiently at the beginning of a lesson.

The review of the previous day's lesson should begin with a brief summary by the teacher. Several sentences that briefly and concisely remind students of what they did and why, and demonstrating how to solve a single problem is usually sufficient. Next comes the checking of homework. This should proceed very quickly once students learn that when math period begins they are to have their homework on top of their desks ready for checking. Initially, it may take some time to establish this routine, but once the routine is established it should take only a couple of minutes to check homework.

The third activity, mental computation, plays two roles in the lesson structure. First, it is an important activity per se (see earlier section). Second, these activities can provide a smooth transition for getting students engaged in thinking about math prior to the point at which the teacher begins a new development lesson.

The ending of the lesson is a very simple procedure. After allowing students a period of time for uninterrupted practice, the teacher briefly checks pupils' work on a few problems (may call on students, ask students who got problems correct to raise their hands, etc.). This accountability procedure encourages students to apply themselves during seatwork and allows an additional opportunity to clear up misunderstanding. After checking some of the seatwork, the teacher ends the mathematics lesson by assigning the homework problems.

The predevelopment phase of the lesson should take roughly eight minutes. The exact distribution of time on review, homework, and mental computations depends upon a variety of conditions (e.g. moderate difficulty with homework vs. no difficulty) and you are asked to use your judgment. In general, we think the following situation will be most applicable: 1-2 minutes on review; 3-4 minutes checking homework; and 3-4 minutes on mental computations.

IX. Summary and Integration

We have asked you to do several things during the next few weeks in an attempt to improve student performance in mathematics. In the first part of this handbook we emphasized that we didn't feel that changing one or two teacher behaviors would make much difference in student performance. We feel that the systematic application of all the behaviors discussed in this treatment program can make an important difference in student learning. The purpose of this last section is to briefly review the teaching requests we have made and to show how each of the pieces fit together into a total program.

The predevelopment portion of the lesson begins with a brief summary and a review of the previous lesson. The review (including the checking of homework) is designed to help students maintain conceptual and skill proficiency with material that has already been presented to them. Mental computation activities follow and provide an interesting bridge for moving into the new lesson.

Next comes the development part of the lesson which is designed to help students understand the new material. Active teaching helps the student comprehend what he is learning. Too often students work on problems without a clear understanding of what they are doing and the reasons for doing it. Under such conditions, learning for most students will be filled with errors, frustration, and poor retention. If student learning is to be optimal, students must have a clear picture of what they are learning; the development phase of the lesson is designed to accomplish this understanding.

The controlled practice that occurs toward the end of the development portion of the lesson is designed to see if students are ready to begin seatwork. It simply doesn't make sense to assign seatwork to students when they are not ready for it. . .practicing errors and a frustrating experience guarantees that student interest and performance in mathematics will decline. The controlled practice part of the lesson provides a decision point for the teacher. If students generally understand the process and are able to work problems correctly, then the teacher can proceed to the seatwork portion of the lesson. If student performance on problems is relatively poor, then the development must be retaught. If students are ready to do practice work, it is foolish to delay them; similarly, if students are not ready to do development work, it is foolish to push them into it. The controlled practice part of the lessons allows the teacher to decide if it is more profitable to move to seatwork or to reteach the development portion of the lesson.

Hence, when teachers move to the seatwork portion of the lesson, students should be ready to work on their own and practice should be relatively error free. Seatwork provides an opportunity for students to practice successfully the ideas and concepts presented to them during the development portion of the lesson. If teachers consistently present an active development lesson and carefully monitor student performance during the controlled portion of the lesson, then student seatwork will be a profitable exercise in successful practice.

The seatwork part of the lesson allows students to organize their own understanding of concepts (depend less upon the teacher) and to practice skills without interruption. The seatwork part of the lesson also allows the teacher to deal with those students who have some difficulty and to correct their problems before they attempt to do homework. If teachers actively monitor student behavior when seatwork is assigned and if they quickly engage them in task behavior and maintain that involvement with appropriate accountability and alerting techniques, then the essential conditions have been created for successful practice.

Homework is a logical extension of the sequence we have discussed. During the mathematics lesson students learn in a meaningful setting. During seatwork students have a chance to practice and deal with material they understand. The homework assignment provides additional practice opportunity to further skill development and understanding.

The above aspects of the mathematics lesson combine to give the student: (1) a clear understanding of what they are learning; (2) controlled practice and reteaching as necessary to reinforce the original concepts and skills; (3) seatwork practice to increase accuracy and speed; and (4) homework assignments which allow successful practice on mastered material (distributed practice which is essential to retention).

To maintain skills it is important to build in some review. Skills not practiced and conceptual insights not reviewed from time to time tend to disappear. Even mature adults forget material and forget it rapidly. For this reason we have asked you to provide for review of material presented the previous week each Monday and to provide a comprehensive review every fourth Monday. Such procedures will help students to consolidate and retain their learning. Finally, we have suggested that the systematic presentation of mathematics material may facilitate student learning (i.e., initial acquisition) such that you can pick up the pace a bit and we encourage you to do so if you can. Finally, when many students experience trouble, the development portion of the lesson should be repeated and students should never be asked to do homework until they are ready to do it successfully.

The plan described above is summarized in Table 2 that follows. This table outlines the sequence and length of each lesson component in order to provide a general picture of the mathematics lesson that we are asking you to teach.

Appendix A

Process/Product Questions

Variable Description

Process questions ask the student to explain something in a way that requires him or her to integrate facts or to show knowledge of interrelationships. Process questions often begin with why or how and can't be answered with one word. Many process questions require the student to specify the cognitive and/or behavioral steps that must be gone through in order to solve a problem or come up with an answer. Two examples of process questions follow. "Allen, if a man bought 3 tickets for $2.85 and 2 tickets for $2.15 and if we wanted to know the average cost per ticket, how could we get the answer?" Similarly, if the teacher asks "There are 60 minutes in an hour, how can we find out how many minutes in ¼ hour?", she or he is asking a process question. The student is asked to explain a process and to verbalize understanding ("we can always find ¼ of anything by dividing by 4 . . .").

Product questions only require a knowledge of a specific fact and can often be answered with a single word or by providing a number (answer to a problem). Product questions often begin with the words who, what, when, where, how much, how many, etc. A written example of a product question would be 7 + 3 = ___? An oral product question would be "zero times seven equals how much?"

Product questions can be transformed into process questions by asking for an explanation rather than an answer. "Why does zero times seven equal zero?" The child is being asked to show awareness of the principle by saying something like "when zero is a factor the product is zero" or "zero times anything equals zero." A written example of a process question would be "7 + 3 = 10 and 3 + 7 = 10, why?" The student is expected to respond with something like "changing the position (order) of the addends (numbers) does not change the sum."

In summary, product questions are those questions that ask students to provide the right answer (how much, what, when). In contrast, process questions ask students to explain how an answer was or could be obtained (why questions).

Problem

Often when teachers think about development and conceptual work, they equate it with process questions. This is not the case. Indeed, often process questions are overused or used inappropriately. The problem with process questions is that they are sometimes ambiguous to the student (what is the teacher asking me?) and may produce an ambiguous student response even though the student understands the concept. Process questions often consume a lot of instructional time (student thinks, mentally practices the response, makes an oral response). Hence, if process questions are overused, a lot of instructional time can be wasted. If selectively used, process questions can be very valuable. For example, by asking a few process questions, teachers can see if students understand the rationale or principle upon which computational work is based and help consolidate student learning.

If teachers are alert to student responses, hold students accountable by asking individual students questions, and keep all students involved in the lesson, then the learning of unproductive habits is minimized. If process and product questions are used appropriately, then student involvement and achievement are enhanced. If they are used inappropriately, then much instructional time is lost and errors are practiced--errors that subsequently are very hard to correct.

Request for Teaching Behavior

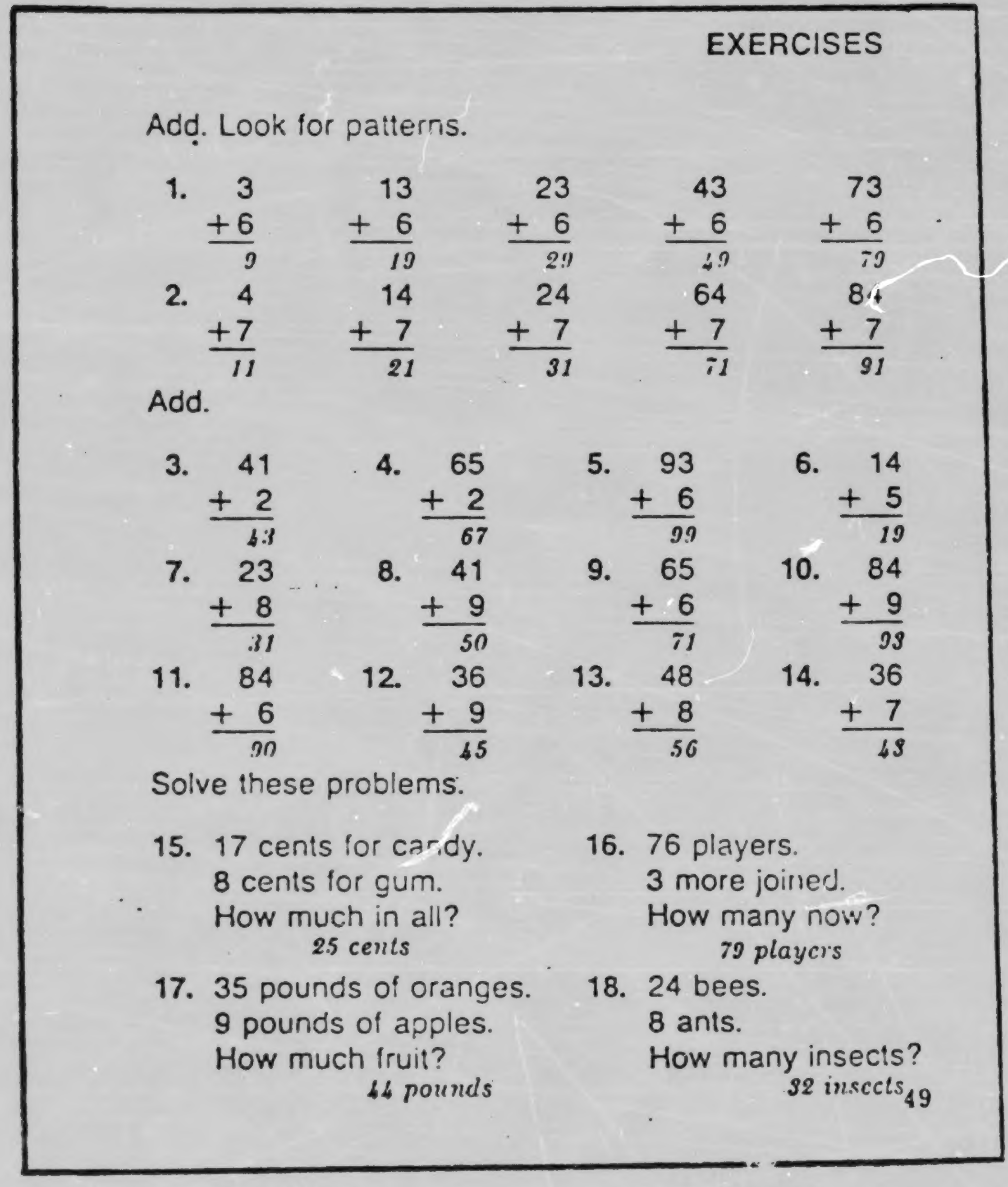
We feel that the presence of a few process questions in the development stage of a lesson are helpful (especially when a new principle is being introduced) because listening to a student's explanation can help teachers diagnose inappropriate assumptions, etc., that students have made. However, we believe that most of the process development can be done through teacher modeling of process explanations rather than by asking students to respond to process questions. For example, the teacher could ask, "Who can tell me what zero times seven is?" The teacher surveys the room and calls upon Bill (who may or may not have his hand up). When Bill says "zero," the teacher could respond with something like, "That's right, Bill, the answer is zero. Whenever zero is a factor, the product is always zero." By actively verbalizing and demonstrating (e.g., writing problem solutions on the board, etc.), teachers can help students to achieve process understandings in a very efficient way. Still, it is useful to ask process questions occasionally to assess student understanding. However, if asked properly, product questions can provide information that assesses the student's ability to relate ideas, transfer concepts to different situations, and understand the process sufficiently well to solve problems. Product questions can also provide all students in the class (or group) a chance to practice the computation. This is especially true when the teacher asks the question first and then calls on a student. If a teacher names a student and then asks the question, many of the students will not perform the calculation (that's Mary's problem). Similarly, if teachers hold non-volunteers accountable on occasion, it increases the number of students who are likely to think about the problem under discussion.

Although a major goal of the development portion of the lesson is to strengthen students' conceptual understanding (why), this goal can be achieved with a heavy use of product questions. The usefulness of product questions is due to the following factors: (1) they typically elicit a quick response from the student (and quick feedback from the teacher); hence, more material can be covered in a given amount of time; (2) they provide more practice opportunity for a broader number of students; hence, a teacher's diagnosis is not limited to the responses of a few students; (3) and they help to create a "can do" attitude on the part of students (a series of quick questions that the students respond to successfully). However, it is desirable to ask process questions and enter a diagnostic cycle (reteaching) when students respond to product questions incorrectly. When students miss the same type of product questions, then it is useful to stop and review the process and ideas behind the computation. To reiterate, process questions can and do play a valuable role in successful mathematics teaching although they should not be overused.

Appendix B

Typical Homework Assignment

Reproduced below is a page from the fourth grade Holt Mathematics textbook. An appropriate homework assignment would be to assign problems #4-18 (evens). The remaining problems could be used in connection with the development or seatwork portions of the lesson. Appendix E shows how these same problems could be put in a different format and thus provide some variety in your assignments.



EXERCISES

Add. Look for patterns.

1.	3 +6 *9*	13 + 6 *19*	23 + 6 *29*	43 + 6 *49*	73 + 6 *79*
2.	4 +7 *11*	14 + 7 *21*	24 + 7 *31*	64 + 7 *71*	84 + 7 *91*

Add.

3.	41 + 2 *43*	4.	65 + 2 *67*	5.	93 + 6 *99*	6.	14 + 5 *19*
7.	23 + 8 *31*	8.	41 + 9 *50*	9.	65 + 6 *71*	10.	84 + 9 *93*
11.	84 + 6 *90*	12.	36 + 9 *45*	13.	48 + 8 *56*	14.	36 + 7 *43*

Solve these problems.

15. 17 cents for candy.
8 cents for gum.
How much in all?
25 cents

16. 76 players.
3 more joined.
How many now?
79 players

17. 35 pounds of oranges.
9 pounds of apples.
How much fruit?
44 pounds

18. 24 bees.
8 ants.
How many insects?
32 insects

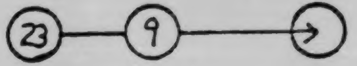
49

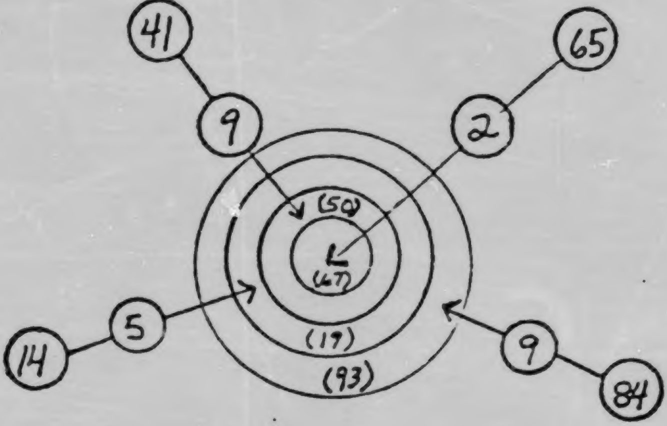
Appendix C

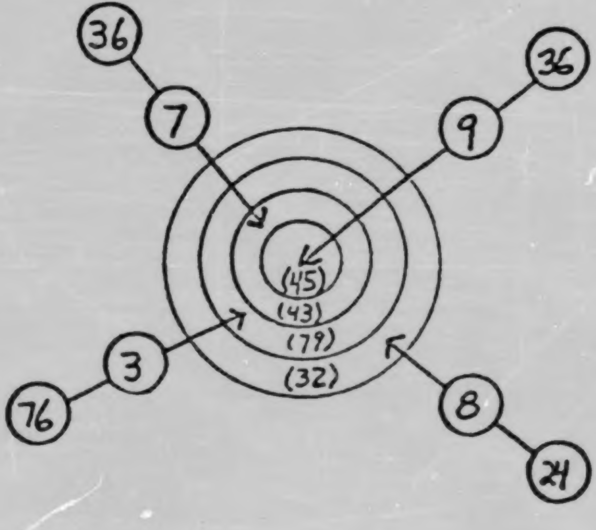
Variety in Assignments

Frequently students can be freed from the somewhat boring routine of always doing problems from the textbook as their homework assignment. The assignment shown below is an alternate to the typical row-by-row set of computation exercises found in most textbooks, yet it accomplishes the same objectives in a more interesting format. Answers for the problems are shown in parentheses.

ADD to find the missing target values. For example, 32 would be the missing value in this example:







Appendix D

Systematic Processing Errors Illustrations

A systematic processing error is an error a student consistently makes on a particular kind of problem. It is different from making random errors. Simple examples include always working addition from left to right or "borrowing" in every subtraction problem whether or not it is necessary. Other common examples are explained below.

In each of the following situations, carefully analyze the examples and try to determine the error pattern. Then check your work by reading the description of the error pattern.

Situation #1

$$\begin{array}{r} 23 \\ +6 \\ \hline 83 \end{array} \qquad \begin{array}{r} 34 \\ +9 \\ \hline 124 \end{array} \qquad \begin{array}{r} 29 \\ +5 \\ \hline 79 \end{array} \qquad \begin{array}{r} 38 \\ +4 \\ \hline 78 \end{array}$$

ERROR PATTERN: In these problems the student does not add straight down a column, but rather adds the number of tens from the first number to the units from the second number. Thus, in example #1 the 2 tens are added to the 6 ones to get 8 tens.

Situation #2

$$\begin{array}{r} 53 \\ -27 \\ \hline 34 \end{array} \qquad \begin{array}{r} 86 \\ -39 \\ \hline 53 \end{array} \qquad \begin{array}{r} 95 \\ -27 \\ \hline 72 \end{array} \qquad \begin{array}{r} 31 \\ -19 \\ \hline 28 \end{array}$$

ERROR PATTERN: In these problems the student does not "borrow," but rather always subtracts the smaller digit from the larger digit.

Situation #3

$$\begin{array}{r} 7 \\ 48 \\ \times 59 \\ \hline 432 \\ 270 \\ \hline 3132 \end{array} \qquad \begin{array}{r} 5 \\ 49 \\ \times 36 \\ \hline 294 \\ 177 \\ \hline 2064 \end{array} \qquad \begin{array}{r} 3 \\ 86 \\ \times 45 \\ \hline 430 \\ 354 \\ \hline 3970 \end{array} \qquad \begin{array}{r} 5 \\ 67 \\ \times 28 \\ \hline 536 \\ 174 \\ \hline 2276 \end{array}$$

ERROR PATTERN: The first part of each problem, the multiplying by the ones is done correctly. However, when multiplying by the tens the crutch number recorded from the multiplying by ones is incorrectly used again. For instance, in the first example, when multiplying by the 5 tens the 7 (carried over from the 9x8) is used again when the 7 is added to 5 times 4 and the 27 is recorded.

Situation #4

$$\begin{array}{r} 4 \\ 26 \\ \underline{\text{x}7} \\ 422 \end{array} \qquad \begin{array}{r} 1 \\ 83 \\ \underline{\text{x}5} \\ 455 \end{array} \qquad \begin{array}{r} 3 \\ 38 \\ \underline{\text{x}4} \\ 242 \end{array} \qquad \begin{array}{r} 2 \\ 53 \\ \underline{\text{x}8} \\ 564 \end{array}$$

ERROR PATTERN: In these problems the crutch is added before multiplying in the tens place, whereas the correct procedure is to multiply and then add the crutch. Thus, in the first example the 4 is added to the 2 and then this sum multiplied by 7. If this problem was done correctly, the 2 is multiplied by the 7 and then the 4 is added.

Situation #5

$$\begin{array}{r} 44 \\ 2/\overline{88} \\ \underline{80} \\ 8 \\ \underline{8} \end{array} \qquad \begin{array}{r} 14 \\ 4/\overline{164} \\ \underline{160} \\ 4 \\ \underline{4} \end{array} \qquad \begin{array}{r} 87 \\ 3/\overline{234} \\ \underline{210} \\ 24 \\ \underline{24} \end{array} \qquad \begin{array}{r} 39 \\ 5/\overline{465} \\ \underline{450} \\ 15 \\ \underline{15} \end{array}$$

ERROR PATTERN: These problems are worked correctly except that the quotient figures are written from right to left. Consider the third example, there are 7 threes in 23, but the 7 is recorded at the extreme right, rather than above the 3.

Situation #6

$$\begin{array}{r} 32\text{r}3 \\ 9/\overline{2721} \\ \underline{27} \\ 21 \\ \underline{18} \\ 3 \end{array} \qquad \begin{array}{r} 78\text{r}2 \\ 6/\overline{4250} \\ \underline{42} \\ 50 \\ \underline{48} \\ 2 \end{array} \qquad \begin{array}{r} 94\text{r}2 \\ 6/\overline{5426} \\ \underline{54} \\ 26 \\ \underline{24} \\ 2 \end{array}$$

ERROR PATTERN: In these problems, whenever the students brings down and cannot divide, he brings down again but forgets to record a zero in the quotient.

Appendix E

Letter to Parents

August 25, 1977

Dear Parent(s):

As part of the fourth grade math instructional program this year, I will be regularly assigning some work for the students to complete at home. It should take your son or daughter about fifteen minutes to complete this homework. If you find that it regularly takes considerably longer for him/her to finish this assignment or the assignment causes other difficulties, please let me know in that I may be assigning too many or too difficult problems.

Programs in other school districts, educational research, and common sense indicate that the more a student practices important math concepts and problems, the more proficient he becomes in essential math skills. I view homework as an opportunity for the student to practice the concepts and skills that he/she has learned in class. I hope that you will encourage your son or daughter to complete every assignment to the best of his/her ability. Parental support is very helpful. Thank you for your cooperation in this matter.

Sincerely,

Appendix F

Teaching Groups in Schools Using a Departmental Organization

The emphasis thus far has been placed upon teaching mathematics to the class as a unit. We feel that many of the principles presented (the importance of development, the use of controlled practice and seatwork, accountability, etc.) will transfer to classrooms in which teachers are teaching groups of students. In applying these principles to a group situation, teachers will have to *adjust* them to their teaching situation.

In general, we are not enthusiastic about the use of two or more groups to teach mathematics. Three recent and major research projects have shown that third, fourth, and fifth grade students appear to benefit more from whole class instruction than they do from individual or group instruction. Although the precise reasons for these differences are unknown, we suspect that students learn less in group and individual settings because they have less direct developmental work with the teacher. Also, the extra transitions (teachers moving from group to group) probably results in the loss of time that could have been used for instructional purposes. Furthermore, student work is probably less effective when the teacher is not available to supervise work.

If the differences between groups are not great, we strongly recommend that the class be taught as a whole class. However, we understand that sometimes the differences between students in a given classroom are so great that grouping is a practical *necessity*.

If grouping is necessary, you should attempt to limit yourself to only *two* groups because the transition and supervision problems that accompany the use of more than two groups are normally very difficult to justify.

Since teaching circumstances are so varied (sometimes the difference between two groups is moderate but in other classrooms there are vast differences between the two groups), it is impossible for us to describe a plan that would be best in all situations. Still, there are a few key things that we would like to emphasize.

First, whenever possible, we think it will be useful for you to teach the class as a group. Students learn a great deal from teacher illustrations and explanations. Perhaps the easiest way to do this in a group situation is by holding common reviews from time to time. The review might be a short-term review for the lowest group and a long-term review for the highest group.

An especially good way to conduct a common review is through the use of mental computation problems. We strongly recommend that each day of the week but Monday you use the first ten minutes of the class for review with mental computation problems. As we have noted earlier in the handbook, we feel that mental computation problems are a very important addition to an instructional program.

Second, we would like you to set aside each Monday for a review session. After spending the first five minutes on mental computation, review ideas and skills that are needed by both groups. Then involve one group in a seatwork review, then begin the developmental review with the other group. Roughly half way through the period reverse the roles; give group two a seatwork review assignment and begin an oral review with group one.

To maximize the value of this review, a homework assignment containing review problems should be given the previous Thursday. Your analysis of these papers should suggest the topics and skills that should receive emphasis in the Monday review. Besides the homework assignment each Thursday, we request that you assign homework three other days per week. Remember that these assignments are to provide brief, successful practice.

The third request is that you maximize the amount of development time for each group. The exact amount to be given to each group will necessarily vary depending on the topic being considered and the group itself; however, the importance of development work for both groups cannot be overemphasized. As you do the development work, remember the guidelines previously discussed. For instance, teacher explanations and illustrations are important, especially initially. Also, process explanations are very important and often times are related to efficient use of limited instructional time.

Finally, we ask that you implement other recommendations as regularly and consistently as you can. Little things are important (e.g., getting all students started on seatwork before doing other instructional tasks) and we hope you will carefully review the ideas presented in the handbook with an eye toward applying them in your classroom.

WEEKLY LESSON TIME TABLE

Monday	Tuesday	Wednesday	Thursday	Friday
Weekly Review (20 Min.) * * * *	Homework, * Review, Mental Computation (8 Min.)	Homework, * Review, Mental Computation (8 Min)	Homework, * Review, Mental Computation (8 Min)	Homework, * Review, Mental Computation (8 Min)
	Developmental (20 min.) * *	Developmental (20 min.) * *	Developmental (20 min.) * *	Developmental (20 min.) * *
Development (10 Min.)	Seat Work (15 min.)	Seat Work (15 min.)	Seat Work (15 min.)	Seat Work (15 min.)
Seat Work (10 Min.)				
Lesson Conclusion & Homework Assign. (2 Min Max)	Lesson Conclusion & Homework Assign. (2 Min. Max)	Lesson Conclusion & Homework Assign. (2 Min. Max)	Lesson Conclusion & Homework Assign. (2 Min. Max)	Lesson Conclusion (2 Min Max.)

* LESSON TIME TABLE (4th WEEK)

Monday	Tuesday	Wednesday	Thursday	Friday
Monthly Review (45 min.) * * * * *	* Review, * Mental Computation (8 Min)	Homework, *Review, * Mental Computation (8 Min)	Homework, *Review, * Mental Computation (8 Min)	Homework, *Review, * Mental Computation (8 Min)
	Developmental (20 min.) * *	Developmental (20 min.) * *	Developmental (20 min.) * *	Developmental (20 min.) * *
	Seat Work (15 min.)	Seat Work (15 min.)	Seat Work (15 min.)	Seat Work (15 min.)
	Lesson Conclusion & Homework Assign. (2 Min. Max)	Lesson Conclusion & Homework Assign. (2 Min. Max)	Lesson Conclusion & Homework Assign. (2 Min. Max)	Lesson Conclusion (2 Min. Max.)

Appendix 2*

Student Types and Effects of Mathematics Program

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* For more details, the reader can consult Beckerman (1981).

General Purpose

The results reported here are from Dr. Terrill Beckerman's doctoral research (1981), and the interested reader can consult his dissertation for an extended discussion. This study was similar to Ebmeier's dissertation (1978), which described the main and interactive effects of a treatment with derived student and teacher types. In the Ebmeier study students stated preferences for a variety of contextual and personal situations, including the amount of structure desired in math and the type of classroom environment preferred. These preferences were used to derive student factors. Students were classified according to factor analyses of student self-report information, and significant main and interactive effects were found for student types and treatment effects.

Beckerman used the same sample in his study (the same treatment was administered, the same observational data were available, etc.); however, in his study, student types were identified by classroom teachers. His research was based upon studies that had examined teacher attitudes toward students (Jackson, Silberman, and Wolfson, 1969; Silberman, 1969; Power, 1974; Good and Brophy, 1972).

Jackson et al. (1969) formed four student typologies from teachers' previous descriptions of students: attachment, indifference, concern, and rejection. Silberman (1969) asked teachers to identify one student whom they felt represented each of Jackson's student types. He observed the interactions of ten third-grade teachers with the students they had categorized. Each class was observed for twenty hours, and the analysis of the data suggested that teachers interacted differently with each of the student types.

Good and Brophy (1972) replicated and extended the work of Silberman with observational research on these four types of students in first-grade classrooms. Their results were similar to those obtained by Silberman. Silberman had collected his attitudinal information before observation; whereas Brophy and Good collected theirs after classroom observation. Apparently teacher behavior was not influenced by the fact that teachers in the Silberman study were asked to classify their students before the study. Good and Brophy also obtained differential teacher interactions with the student types in schools having different SES population. The patterns of teacher interactions with the four types of students appeared to be independent of school context.

Power (1974) conducted a study of the effects of teacher-student interaction, student characteristics, achievement, and attitudes. A battery of tests measuring twenty-three pupil cognitive, instructional, and personality characteristics was administered to 150 grade A students. A second battery of tests which measured ten outcome variables (including achievement, attitudinal, and sociometric variables) was administered at the end of the school year. Through a series of canonical analyses of the ten outcome variables with the twenty-three pupil characteristics, four student types emerged: success, rejection-dependency, person-orientation, and social orientation.

Three of Power's derived student types closely corresponded to three of the student types reported in the Jackson et al. study. Jackson's rejected student is similar to Power's alienated student; both of these student types are seen as overwhelming and frequent causes of behavioral problems within the class. Power's dependent student is similar to Jackson's concern student, and Jackson's attachment student and Power's success student also have like characteristics.

Good and Power (1976) attempted a synthesis of the work of Jackson, Silberman, and Power. They added a fifth student type (phantom) to the four student types researched earlier (Jackson, Silberman and Wolfson, 1969; Silberman, 1969; Good and Brophy, 1972). It appeared that the types of students described by teachers and the types of students derived by examining student preferences, achievement, and personality characteristics could be summarized into five types. A brief description of these five types of students, based upon the work of Good and Power (1976), follows:

Student Types:

1. Success students. These students are essentially task-oriented and academically successful. They are cooperative in class, tackle almost all questions, and create no discipline problems. The teacher is more likely to direct more difficult questions to them, and they get most of them right. Success students like school and tend to be liked by both teachers and peers.
2. Social students. These students are more person- than task-oriented. They have the ability to achieve, but value friendship more than school work. They are likely to get called on fairly often by the teacher to help them become involved and because they are able to answer easy questions. However, some of their answers are incorrect or irrelevant. Also, social students are among those most likely to be criticized by the teacher. While they are fairly popular and have many friends, some social students are not well-liked by their teachers.
3. Dependent students. These students are the clinging vines of the classroom, always looking for teacher support and encouragement, asking for direction and help. They are frequent hand-raisers, more likely to guess and make errors, and make extensive but roughly task-appropriate demands on the teacher. Most of these

students achieve at a low level. Teachers generally express concern regarding dependent students while their peers reject them.

4. Alienated students. These students include the disadvantaged and the reluctant learners. In the extreme, they reject the school and everything it stands for. This rejection may take one of two forms: open hostility or withdrawal. It follows that they are either highly aggressive and create serious behavior problems or they withdraw to the fringes of the classroom and are ignored by the teacher entirely. Teacher attitudes usually reflect rejection or indifference.
5. Phantom students. In most instances, these students are neither seen nor heard in the classroom. They are about average on everything but involvement in public settings. Some of them are shy, mousy students while others are quiet, independent workers of average ability. They are rarely actively involved in class or group activities, never volunteer, and never create problems. The teacher will have trouble remembering who they are and express attitudes of indifference toward them, as will their peers.

Approximately ten weeks after school began, control and treatment teachers were given Power's student descriptions and were asked to classify their students. In addition to the five typologies, a sixth classification was included for students who did not fit any of the other five descriptions. Only fourteen students out of a sample of over 500 were classified as not belonging to one of the five types. All teachers except two felt that the five types were quite appropriate for describing students. Two teachers who said they had difficulty classifying students were dropped from the sample in an effort to include only those teachers who felt that they understood the rating form.

Interaction of Treatment With Student Types

A three-way, completely crossed-factorial design was used for the analysis. The independent variables were treatment, student type, and sex. The treatment variable had two levels: one level represented the math program (Good et al., 1977); the second level represented the control group. The student classification had five levels, each representing a different student typology. Sex was included as a control variable, to determine whether the treatment of student types interacted with the sex of the students in the study. Separate three-way models were used for analysis of the two dependent variables, the measure of student achievement (residual SRA score), and the student attitude measure. It is beyond the scope of this report to fully interpret and discuss the data, although much of the relevant data is presented in Tables 1-4. Again, the interested reader is referred to Beckerman (1981) for further details and discussion.

As can be seen in the tables, there were significant main effects for treatment types and student types, but not for sex, when the SRA residual scores were used as the dependent measure. All five student types obtained higher mean SRA residual scores in the treatment classrooms than in the control classrooms. It could also be seen that both males and females benefitted from being in the program. A few brief comments about each of the student types follow.

Success students had the most positive achievement gain of the five student types in classrooms implementing the mathematics program. On the basis of these data, it could be concluded that high-achieving, high-ability, independent students have better performance in a highly structured, briskly paced, and teacher-directed learning setting than do other students.

Social students in the treatment program also had greater gains than were predicted and also significantly more positive attitudes than social students in control classrooms. Beckerman suggests two possible explanations for these results. He argues that the highly structured, formal setting led social students to have more successful academic experiences than they had in other educational settings. Also, the teachers' checking and recording of students' successful performances further motivated students to continue academic work. He suggests that a second explanation for social students' improved performance and more favorable attitudes relates to the high participation aspects of the treatment. He writes, "The mathematics program advocated frequent teacher-student interaction throughout the review and development sequences. The mental computation exercises which occurred within the review sequence were conducted totally through verbal interaction. A general criticism by this investigator/observer concerns the mental computation exercises. Classroom observation of this exercise indicated that generally only about half or less of the students were actively participating, because the questions were too difficult for the majority of students to solve quickly. For the high achiever in general and the social student, who prefers to interact, this would be a positive situation. Perhaps a social student frequently interacts in non-academic ways, because a typical classroom does not provide enough opportunity for them to interact in academic ways. The positive correlational finding for mental computation and review (though not significant) combined with overall analysis supporting the program in general, support the hypothesis that the high participation aspects of the program was particularly beneficial for social students." (p. 112).

Beckerman also provides a brief rationale for the effects of the program on dependent, alienated, and phantom students. This material follows. "The dependent student type had both better and slightly higher than predicted performance in classrooms implementing the program. This student type was described as a very conforming, highly anxious, low-achieving student who makes frequent demands upon teachers for individual help and/or reinforcement. Prior research indicated that this type of student performed well in high-structured, high-participation settings where the objectives were clearly organized and described. In particular, results from the Whitzel and Winne (1976) and Bennett (1976) studies indicated that low-achieving students had better performance in an individualized setting with the objectives being matched to the individual's ability and the environment was student-centered. Perhaps a brisk pace and objectives were more appropriate for higher achievers. However, the homework assignments (significantly correlated) provided additional practice needed to comprehend objectives and to keep up with the brisk pace maintained in the classroom.

In general, the program is a very positive learning environment for the dependent student type. However, teachers should be cautioned not to increase the pace in interactions with other student types at the expense of the dependent student. That is, teachers should maintain a brisk pace, but also provide increased amounts of individual teacher-student interaction with the dependent student.

The alienated student type had greater performance in program classrooms than in control classrooms, but still performed much lower than predicted. The program was least successful with this type of student. Prior research, particularly the Bennett (1976) findings, indicated this type of student has better performance in an individualized setting, where the curriculum and objectives are matched to the individual's pace and the program is student oriented (i.e., affective/aesthetic).

Homework was positively related to the achievement of the alienated student. That is, homework is advantageous and enhances the achievement of the student type. However, the negative residual score, and negative correlations of the observational data indicate program modifications are necessary to significantly improve the performance of the alienated student type. These findings in conjunction with prior research findings (Bennett, 1976; Ebmeier and Good, 1979; Solomon and Kendall, 1976) suggest that the program should be more individualized for this student type. Perhaps individualized workbooks and assignments, in connection with more opportunities for the student to work independently (i.e., less teacher interaction) will provide a beneficial environment.

Interestingly, the phantom student had better performance in the mathematics program classroom than in the control classrooms, yet significantly more negative attitudes. However, while achievement was better in the program classrooms, it was still slightly less than predicted. The low residual score and negative attitude findings suggest that the mathematics program should be modified to some extent. The phantom student was, in part, described as an independent student of average ability/achievement. Relevant findings from prior research (Peterson, 1976; Bennett, 1976; Solomon and Kendall, 1976) described a high-structured, low-participation, formal settin as a more beneficial environment, compared to individualized, informal or high participation settings. In general, the mathematics program was described as a high-participation, teacher-directed, formal-instructional system. In particular, the program called for frequent teacher-student interaction and accountability behaviors. Relative to the phantom student, the encouragement of frequent teacher-student interaction possibly had a more detrimental influence, particularly upon their enjoyment of the math class (i.e., their attitude), than a positive influence.

Correlational results indicated the checking of homework was possibly associated with achievement of this student type. The general implication of the findings may be that the phantom students prefer teacher recognition of their performance but not through public interaction (this material was drawn from pages 113-115 in Beckerman's dissertation).

Direction for Future Research

These results, along with those presented earlier by Ebmeier (1978) and Ebmeier and Good (1979) indicate that the mathematics treatment program we experimented with was generally successful. That is, students in all cases made more gains in the treatment classrooms than they did in the control classrooms. However, in some cases, these were important beneficial gains but for some types of students (particularly in combination with certain types of teachers), the gains were not particularly important or impressive. We feel that what is needed at this point in time is a very comprehensive, analytical examination of the interactions of different types of programs with different types of students. The field has started to produce some interesting experimental programs, but we need to begin to integrate and consolidate these findings across studies and look for their implications for teaching. An example of what this integration might ultimately look like is provided in a brief table that Beckerman prepared (see Table 5). To reiterate, what we need now is a very careful integration of available results and then to begin to devise studies that experimentally test some of the hypotheses that emerge from this synthesis. Performing this syntehsis will be a very difficult, analytical task because treatments vary in their composition from study to study and also definitions of student types use different operational procedures. Nonetheless, it seems important to begin such work prior to the collection of new data efforts.

It will not be easy to make detailed comparisons across different studies for a variety of reasons. For example, in Beckerman's research (1981), the teachers who classified students according to the five types assigned students disproportionately. In particular, 249 students were described as success students, 147 students were defined as social students, 90 students were defined as dependent students, 43 students were described as alienated students and 69 students were described as phantom students. Teachers in different populations might assign more or less students to particular cells, and indeed even in this study teachers showed individual differences in the number of students that they assigned to the five types. Although such categories are helpful in getting the phenomenological reactions of teachers to students, they also make it likely that definitions will vary from teacher to teacher and sample to sample, depending upon the relative standards that teachers impose.

In future research, it may be useful to have teachers use some sort of distribution for classifying students. Also, it may be useful to experiment with having teachers identify only two or three students per category in order to keep the typology as pure as possible. Given that teachers' definitions in defining students and the criteria they impose in assigning students to cells may vary from teacher to teacher, it is very difficult to tell how closely teacher-assigned categories parallel typologies derived using other sources. Indeed, treatment variations vary widely from study to study and what one investigator calls open may be relatively structured in a different population. Also, when data are broken down by achievement level, what is a group of high-achieving students in one study may correspond more closely to the absolute achievement level of students classified as medium achievers in other work. Again, such comparisons will not be easy but it seems that the field is ready for some synthesizing that will perhaps generate hypotheses that can be tested in the future and such synthesizing work may also lead researchers to begin to impose a more standard and more rigorous definition of the populations they are working with.

Table 1

Analysis of Variance
Dependent Variable--SRA Residual Scores

Source	df	SS	F	p
Treatment	1	838	26.10	.001
Student Type	4	879	6.84	.001
Sex	1	62	1.93	.17
Treatment x Student Type	4	45	0.35	.84
Treatment x Sex	1	14	0.44	.51
Student Type x Sex	4	19	0.15	.96
Treatment x Student Type x Sex	4	62	0.48	.75
Error	578	188558		

Table 2

Mean [illegible] Residual Scores for the Three Independent Variables--Treatment Type, Student Type and Sex Type

Variable	n	Mean Score
Treatment	370	1.42
Control	228	-2.00
Student Type 1	249	1.52
Student Type 2	147	-0.27
Student Type 3	90	-0.45
Student Type 4	43	-3.12
Student Type 5	69	-1.38
Males	506	0.52
Females	292	-0.27

Table 3

Differences of SRA Residual Scores for Student Types in Treatment and Control Classrooms

Student Type	Treatment $\overline{X}$	Control $\overline{X}$	Difference
Success	2.72	-0.60	3.32
Social	0.99	-2.97	3.96
Dependent	0.56	-1.96	2.52
Alienated	-1.14	-4.68	3.54
Phantom	-0.37	-2.61	2.54

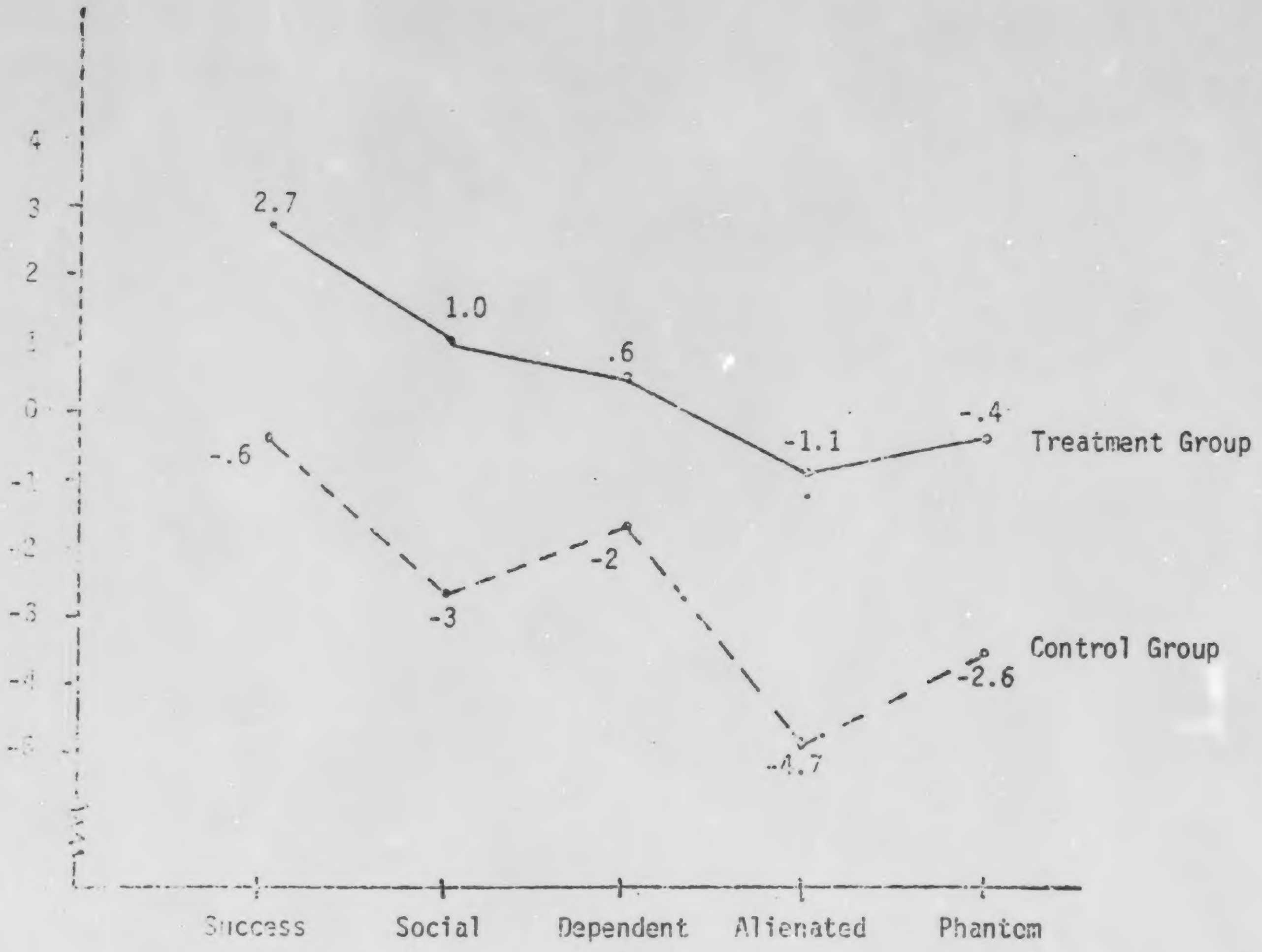

Table 4. Comparison of Mean Residual SRA Scores for Five Student Types in Treatment and Control Classrooms

Table 5

CHARACTERISTICS	AUTHORS	MOST BENEFICIAL ENVIRONMENT
Success Student		
1. High Achiever/High Ability	(Whitzel and Winne, 1976)	Fast paced setting, high structure, low or high participation, whole class or traditional approach as opposed to an individualized setting.
2. Independent/High Achiever	(Peterson, 1976)	
3. Type 1	(Bennett, 1976)	
4. Cluster 1 and/or Cluster 5	(Solomon and Kendall, 1976)	
Social Student		
1. High Ability	(Whitzel and Winne, 1976)	Fast paced setting, high structure, high participation, whole class or traditional approach as opposed to individualized, student centered, approach.
2. High Ability	(Peterson, 1976)	
3. Type 8	(Bennett, 1976)	
4. Cluster 3	(Solomon and Kendall, 1976)	
5. Type 4	(Ebmeier and Good, 1979)	
Dependent Student		
1. Low Achiever	(Whitzel and Winne, 1976)	High structure, high participation, traditional or formal approach.
2. Low Ability, Conforming High Anxiety	(Peterson, 1976)	
3. Type 4	(Bennett, 1976)	
4. Type 1	(Ebmeier and Good, 1979)	
Alienated Student		
1. Low Achiever	(Whitzel and Winne, 1976)	A low participation, individualized setting, emphasis upon affective/ aesethic rather than cognitive.
2. Low Achiever, Low Anxiety	(Peterson, 1976)	
3. Type 7	(Bennett, 1976)	
4. Cluster 2	(Solomon and Kendall, 1976)	
5. Type 3	(Ebmeier and Good, 1979)	
Phantom Student		
1. Independent, Quiet	(Peterson, 1976)	A low participation, high structure setting.
2. Type 5	(Bennett, 1976)	
3. Cluster 5	(Solomon and Kendall, 1976)	

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Appendix 3

Analysis of Fourth-Grade Mathematics Treatment Program Effects on Students At Different Achievement Levels

The data briefly reported here address the question of differential impact of the fourth-grade treatment on different ability students. The students were divided into quartiles (on the basis of their pre-achievement scores) and that classification was used as an independent variable along with status (treatment or control) with residual achievement serving as the dependent variable.

There was a strong main effect for status (p = .0001). Students at all achievement levels benefitted from treatment versus control status. There was a weak main effect for achievement (p = .08) and no interaction effect (p = .53).

Although the main effect for achievement is very weak (p = .082), it does merit comment. From the examination of the mean scores (see Table 1), the treatment seemingly had the most impact on the slightly below average students. It should be noted that this effect was very weak and also that the analyses reported here are based upon using the student as the unit of analysis (which tends to exaggerate effects). In reporting the treatment effects in the fourth-grade study (see the body of this report for those results...or see Good and Grouws, 1979, Final Report), the classroom was the unit of analysis used for making judgments about the effectiveness of the program as a whole (a more conservative approach). However, here we were looking at the effects of the program on students at different achievement levels across the sample as a whole and using the students as the unit of analysis seems to be an appropriate way to make this judgment.

Such data suggest that the treatment had generally positive impact but that some students benefitted more from the program than did other students. More details of this analysis as well as comparable data for the sixth- and eighth-grade treatment programs will be presented subsequently (Good, Grouws, and Ebmeier, in progress).

Table 1

Students' Residual Mean Scores on the Basis of Student Achievement on the Pre-Test and as a Function of Program Participation (Treatment or Control)

	Treatment	Control	Difference
Top quartile	1.66	-1.53	3.19
Next quartile	.75	-2.01	2.76
Next quartile	2.58	-1.35	3.93
Lowest quartile	1.57	- .95	2.52

Appendix 4

The Verbal Problem Solving Treatment Manual

Verbal Problem Solving Manual

Missouri Mathematics Effectiveness Project (MMEP)

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Verbal Problem Solving

Introduction

There are many reasons for teaching students mathematics and different people stress different reasons as they testify to its importance. On one thing, however, there is universal agreement: mathematical problem solving is of paramount importance! This agreement stems from the fact that many real world problems are most easily solved by expressing and treating them mathematically. An important step toward developing problem solving ability in students is to help them gain competence in solving verbal problems. By verbal problems we mean those problems which are commonly referred to as "story problems" or "word problems." These are the problems that are traditionally found in contemporary student mathematics textbooks.

In the past, instruction on verbal problem solving has amounted to little more than the teacher solving a few sample problems in front of the class and then asking students to solve similar problems on their own. Usually such instruction is grossly inadequate; students do not understand the assignment and are not able to do the problems successfully. Because of such poor presentation many students develop a permanent dislike for these problems. This situation is particularly unfortunate because research has shown that there are a number of instructional strategies that can be used to improve student problem solving performance significantly. The remainder of this manual is devoted to describing techniques that can be incorporated successfully into daily instructional practice. When these techniques are used systematically we believe that students' ability to solve verbal problems will show steady progress.

In particular, it is important to include some work on verbal problem solving each day. Too often verbal problem solving is taught only three or four times a year as a special topic. However, it is only the day to day brief but systematic exposure that will allow students to become proficient in solving mathematical problems.

Problems Without Numbers

The use of problems without numbers is a very effective instructional technique for improving verbal problem solving performance. It provides students an opportunity to gain insight into the problem solving process by avoiding the use of numbers and thus the need to perform any computation whatever.

Example

To illustrate the method consider the following typical problem:

> Two classes sold 100 football game tickets.
> One class sold 27 tickets.
> How many did the other class sell?
> (Holt School Mathematics, Grade 6, p. 32)

This problem can easily be rephrased so that it is a problem without numbers:

> Our class and Mrs. Smith's class sold tickets.
> We know how many tickets were sold altogether
> and how many tickets our class sold.
> How many tickets did Mrs. Smith's class sell?

The teacher presents only the problem without numbers and asks the class how to solve it. An appropriate answer might be something like this: "I'd subtract how many tickets we sold from the total number of tickets to find how many tickets Mrs. Smith's class sold." Time permitting, the teacher should follow-up with another problem without numbers or occasionally consider the same problem only with the numbers included.

Rationale

The specific reasons why this technique is effective are difficult to isolate. One reason for its effectiveness may be that it causes students to focus exclusively on the method needed to solve a problem without any numerical or computational distractions. Many teachers realize that too frequently students begin doing the computation before they have really thought through the problem. In fact, some students have been known to begin computing before they have read the entire problem! Avoiding the use of numbers tends to resolve these kinds of problems. Since the strategy does not require computation, students can be exposed to a substantial number and variety of verbal problems in a short period of time.

Implementation

This technique should be used frequently as part of a comprehensive effort to improve verbal problem solving skills. It seems especially effective if teachers create the problems to be used by recasting verbal problems found in the student textbook. It is also helpful if the problems are written down and ready for presentation prior to the beginning of the math period. This allows efficient use of the available instructional time.

Writing Verbal Problems

Research has shown that when students create and write verbal problems, their problem solving ability improves. Certainly a comprehension of what constitutes a problem is necessary in order to succeed at writing problems, and this is turn may be a vital component in learning to solve verbal problems.

Example

There are a variety of interesting formats that a teacher may use when having students write verbal problems. One method is to supply data and ask students to make up their own problems based on this information. For example, the data might consist of a football team roster like the one below.

Number	Player	Position	Year	Height	Weight
11	Anderson, Bill	Quarterback	9th	5'8"	155
24	Baker, Burt	End	8th	5'7"	140
17	Brunson, Jim	Quarterback	8th	5'5"	135
..	....	....	...	....	...
..	....	....	...	....	...

To illustrate the kinds of problems that may be written, the teacher could supply examples like the following which range from the easy to the complex:

> Bill Anderson and Jim Brunson are both quarterbacks on the Memorial Junior High School football team. Bill weighs 155 pounds and Jim weighs 135 pounds. Bill weighs how much more than Jim?
>
> There are three quarterbacks on the Memorial team. Jim weighs 135, Bill 155, and Sam 130. What is the average weight of the quarterbacks?
>
> All 33 players on the Memorial team are going on the bus to the away game with Fulton Junior High. Highway 24 is the shortest way to Fulton, but the Mason Creek bridge on this route limits loads to less than five tons. The bus with the driver weighs 3200 pounds. Will the bus loaded with the players be too heavy to use Highway 24?

After students have had some experience writing problems, the teacher may allow them to make up problems by supplying their own data from situations that are of interest to them. Placing some restrictions on the problems to be written will help to keep this activity consistent with the operations and kinds of numbers currently being studied. For example, a teacher might want to restrict the problems written to those that can be solved by division of whole numbers or to those involving addition and subtraction of fractions.

Rationale

The value of having students create verbal problems is closely tied to their simultaneous development of the ideas of information given, information to find, and a link or path from the former to the latter. Writing a problem requires attention to all three components. In the early stages of this development a student may only consider the given aspect and write a "problem" like:

> Suzi has 9 packages of baseball cards.
> There are 12 cards in each package.

As students progress in their ability to comprehend what constitutes a problem and thus the ability to write problems, there is likely to be some transfer to those situations where students are presented with problems to be solved. This transfer may be in the form of recognizing what is given, what is to be found, or that the task is to build a bridge or link between the two. The importance of this transfer is emphasized by the number of times we have all heard the comment: "I really don't know where to begin." If teachers regularly have students write verbal problems, they should hear this question much less frequently.

Implementation

This technique can be closely tied to instruction on any of the basic operations (addition, subtraction, multiplication, and division) as well as most other topics, including measurement and geometry. Students may be asked to write problems in class, as part of a homework assignment, or both.

Allowing students to solve one anothers' problems often stimulates their interest. Contests based on ideas like "stump the teacher" and "problem of the week" also add variety and interest.

Estimating the Answer

Students who estimate the answers to verbal problems before they attempt to solve them seem to make important gains in the ability to correctly solve problems. Use of this technique is not difficult, yet the payoff from using it can be substantial.

Example

Students can be asked to estimate the answer to any verbal problem. Consider this problem:

> Janet picked 17 daisies for each of her
> classmates. She had 38 classmates.
> How many daisies did she pick in all?
> (Holt School Mathematics, Grade 6, p. 63)

Students may estimate the answer to be 600 by formally thinking of the product 15 X 40, or by informally thinking of 15 sets of 40. Another estimate might be 700 by thinking that the answer will be somewhat less than 20 X 40. Each of these estimates is close enough to the exact answer of 646 to serve the desired purpose. Of course, students may estimate the answer in an entirely appropriate way that is very different from the formal and informal methods mentioned here. A discussion of the methods used to estimate a particular answer can be very enlightening for students and teachers alike. In particular, such discussions provide an excellent learning experience for those students who have a poor concept of what is involved in the estimation process.

Rationale

The benefits derived from using the estimation strategy may be due to several factors. In order to estimate the answer to a problem a student must comprehend, at least in an intuitive way, what the problem is about. This is an important first step in solving a problem. A reasonable estimate of the solution also suggests and eliminates certain computational procedures. For instance, in the previously cited example the operations of addition, subtraction, and division are ruled out quickly since there is no way they can operate on the numbers in the problem (17 and 38) so that the result will be anywhere close to a reasonable estimate; in fact, such operations would not even yield a three-digit number!

Another factor which may contribute to the value of estimation is that it provides a safeguard from absurd answers and thus provides a means of detecting computation errors. Although there may be other reasons why the estimation technique is so effective, suffice it to say that the results are generally very positive.

Implementation

The estimation technique is easy to use and should be used in two distinct situations. First, it should be used regularly as an instructional method, perhaps by being a part of a regular rotation among other problem solving methods. Second, once students are acquainted with the idea, they should be required to make and record an estimate of the answer for every verbal problem they solve. Teachers are responsible for soliciting and discussing estimates for all problems worked orally in class. They should also monitor seatwork and homework to insure that students are estimating answers in these situations too.

One successful approach to monitoring is to have students record their estimates and then identify them by underlining them. Exact answers are then either circled or underlined twice.

It is important to emphasize again that discussion of the various methods of making an estimate for a specific problem is an ideal learning situation for those students having difficulty with this technique. Teachers can also foster the initial development of this ability by thinking aloud as they make their estimates as part of work done in front of the class.

Providing practice in rounding numbers and doing mental computation is also beneficial. A teacher must emphasize that in order for an estimate to be helpful it must be carefully made and not a "wild guess." Teachers can best do this early in the year by frequently modelling (thinking out loud) and clearly demonstrating to students how to make estimates.

One final thought to keep in mind as you do estimation work is that estimating can be informal in nature and need not rely on formal calculation, either written or mental. Recall that the product of 20 and 40 can be thought of informally as 20 groups of 40, and the approximate result gained from relying on one's quantitative sense is usually accurate enough to serve the desired purposes outlined in this section.

Reading Verbal Problems

The inability to read verbal problems is a definite factor in the difficulty many students have in learning to solve verbal problems. Thus a sustained effort to overcome reading problems is necessary in order to improve verbal problem solving ability significantly.

Example

There are many facets to the reading process that must be taken into account in the instructional process. To read well a student must be able not only to "string words together," but also to comprehend these words. Consider this problem:

> The Great Pyramid was originally 481 feet tall.
> The Great Pyramid was as tall as a building of
> how many stories, if you use 12 feet per story?
> (Addison Wesley, Investigating School Mathematics,
> Grade 6, p. 141)

There are many kinds of reading-related difficulties associated with verbal problem solving. An initial difficulty in the example problem might be with recognition of words like "Pyramid" and "building." Another difficulty, associated with a higher level of thinking, might be recognizing a word but not associating it with its appropriate meaning. In the example problem a student might incorrectly think of the word "stories" as being a collection of narratives rather than a measure of the height of a building. Finally, even if the words and their meanings are correctly discerned there is sometimes difficulty with general comprehension. Among other things the student must realize what information is given and what is to be determined.

Rationale

If a student cannot read a problem he is going to have great difficulty solving it. We now examine a method for handling these reading-related problems.

Implementation

There are two goals to be worked on jointly. First, assistance must be given to students to help them overcome their reading problems. Progress on this goal is oriented toward a long term solution to the problems, which in turn will result in better problem solvers. The second goal is to provide practice in solving verbal problems which circumvent reading difficulties. This is done by the teacher reading problems aloud, using tape recorders, and so on. The second goal insures that improvement in verbal problem solving will not have to wait until the reading difficulties are remediated which in many cases may involve a considerable period of time.

Several things must be done as part of our *regular mathematics instruction* regardless of the particular topic being studied in order to reduce the possibility of later reading difficulties. Terminology must be given special attention. Whenever a new term is introduced it must be written on the board, carefully pronounced first by the teacher then by the students, and then its meaning must be carefully discussed. This discussion should include both examples and nonexamples of the concept and also distinguish between the mathematical meaning of the word and any nonmathematical uses of the word. For example, the word "plane" has a special mathematical meaning quite different from everyday use where it might designate an airplane or a hand tool.

Whenever verbal problem solving is the *main topic for a lesson* the teacher must take direct steps to deal with reading problems. This means that all problems presented in the development part of the lesson and the first problem in any seatwork assignment must be carefully read aloud by the teacher or a student and important words and ideas discussed. An example of how this is done is described later in this section. Students must also be given reading assistance on more than the first seatwork problem. A teacher could effectively make use of audio recordings of the problems, or provide reading assistance as needed and requested during the seatwork time.

Special attention to reading problems alone should be included periodically during the daily portion of the mathematics period which is devoted to problem solving. This may involve teachers and students alternately reading problems, with a discussion of each problem after it is read. For example, in the problem:

> Waves as high as 112 feet have been reported on the "high seas." If each floor of a building is 14 feet tall, the wave would be as tall as a building with how many floors?
> (Addison Wesley, Investigating School Mathematics, Grade 6, p. 109)

several meanings of the word "wave" could be discussed, and attention would also be given to identifying the two pieces of information given and what needs to be determined to solve the problem. Problems to be read may be collected from the textbook, teacher and student written problems, and problems from older textbooks which are no longer in use in the school district. In order to focus primarily on reading, especially reading for meaning, problems read and discussed need not be solved. This allows for many problems to be considered in a short period of time.

Progress on reading difficulties should result from the above mentioned suggestions. Of course, progress can also be expected from students due to their regular reading instructional program. Certainly it is quite appropriate for mathematical material to be used as part of this instruction. Finally, not all students will benefit to the same degree from the attention to reading problems, but it will be a valuable experience for some students.

Anyone who has taught verbal problem solving is aware that reading problems which hinder verbal problem solving do not appear in isolation. How many times have you read a problem to a nonreader and he still could not solve the problem? For this reason attention to reading is only one of the many important techniques that must be given regular attention in instruction on problem solving.

Writing an Open Sentence

Many potential benefits of mathematics learning are realized when mathematics is used to model physical situations, because it is in this way that mathematics is used to solve everyday problems. The simplest situation where this takes place is where an open sentence is written to represent a verbal problem involving a minimal number of conditions. Verbal problems can often be solved without going through this step but some research has shown that developing the ability to translate problem conditions into mathematical sentences is related to improved problem solving performance.

Example

To illustrate how an open sentence can be used to model a verbal problem or a real world situation, consider this example:

> Nine classes in the school gave a total of
> $1,080 to the Book Fund. Each class gave
> the same amount. How much did each give?
> (Holt School Mathematics, Grade 6, p. 87)

This problem can be translated into the open sentence $9 \times \square = 1080$. The answer to the problem is then found by solving the open sentence using informal means such as estimation or by formal calculation of the quotient 1080 divided by 9.

Rationale

As with many other successful techniques, this technique probably gains much of its power by forcing the student to read carefully and to come to grips with the meaning of the problem. That is, it is necessary to determine how the given information pieces relate to one another in order to write an appropriate open sentence. Another reason for the usefulness of this method is that it reduces memory load in complex problem situations. Given a complex problem involving many conditions it is difficult, if not impossible, to mentally remember, manipulate, compare and contrast the given conditions. If on the other hand, these conditions are represented in the form of a collection of open sentences the task becomes much more manageable. For example, the following problem is difficult to solve without the use of open sentences.

> In order to put a fence around two adjacent sides of a rectangular lot 38 feet of wire is needed. The area of the lot is 217 square feet. What is the length of each side of the rectangle?

Let x and y represent the length of the two adjacent sides. Then $x + y = 38$ and $xy = 217$. From the first sentence we know $y = 38 - x$ and when we substitute this into the second sentence we have $x(38 - x) = 217$. This can now be solved by trial and error or more formal means, but in either case getting the problem into manageable form involved writing open sentences (equations).

Implementation

This problem solving method should be taught to all students. Hopefully most students will already have had prior exposure to the technique and thus only periodic review will be necessary in order for them to use the technique as they solve verbal problems. The periodic review can be part of a rotation among other techniques described in this manual and like the others can be done daily using a small part of the mathematics class period.

When providing practice on translating verbal problems to open sentences several important ideas need to be remembered. First, there will be a tendency for students not to write an open sentence for the very simple problems. The typical comment will be "I already know how to do it!" The teacher must persevere and require that sentences be written in most cases because only in this way will the skill be learned and the student develop the capability to apply the skill in more complex situations. Attempts to translate difficult problems involving many conditions to open sentences without considerable practice on simple problems usually ends in failure.

Second, teachers must be aware that several different open sentences may equally well model the same verbal problem. In the example already described the sentence 1080 - 9 = ☐ could be used as a model quite appropriately. It's likely, however, that many students will think of the problem multiplicatively and write the sentence 9 X ☐ = 1080. Either sentence is acceptable and both lead to a correct solution. Finally, make students aware that any one open sentence may model a large number of situations that seem perceptually different but are alike structurally.

Several other suggestions may be of help. When this technique is being used as just a small segment of the lesson (e.g., 10 minutes) it will be desirable frequently to have students only write the open sentence that goes with a problem and not continue to find the exact solution. Also, for those students who have had little work with using open sentences in this way it will be easier if the initial translations involve simple problems. Using problems from textbooks at lower grade levels is often a good idea under these circumstances.

Appendix 5

Initial Reactions of Partnership and "Non-User" Eighth-Grade Junior High Teachers to the Missouri Mathematics Effectiveness Program

Reactions of "Non-User" Teachers to the Project

The evaluation sheets which the twelve teachers brought to the initial project meeting indicated extremely favorable attitudes toward the program. At least eight teachers rated every aspect of the program except pace as either good or very good. The review and development phases received the highest ratings. Although increased pace was valued least, six teachers rated this part of the program as good or very good. There was a good deal of similarity between some aspects of the program and the methods teachers were already using in their classrooms. Daily review (eleven teachers), development (eleven), and teaching the class as a whole (ten) were most often designated as having either great correspondence or general overlap with teachniques teachers were already using. Mental computation, verbal problem solving, and weekly review were used least often by teachers.

Tables 2 and 3 indicate that teachers were familiar with all the verbal problem-solving strategies except problems without numbers. At least seven teachers reported using each verbal problem-solving strategy either frequently or occasionally. Writing an open sentence and estimating the answer were employed most frequently. General strengths of the program which were most often mentioned were its built-in daily, weekly, and broad reviews (ten teachers). Five teachers liked the lesson plan because class time was well-structured and five teachers also liked the verbal problem-solving manual. The weakest aspect of the program, as indicated by seven teachers, was that it was too regimented, did not contain enough variety, and might become boring for students. Six teachers noted that the project required them to do too much paperwork and recording of scores. Five teachers thought there were difficulties with the program because of the 40-minute time limit which they had for each class. It was difficult for them to work all aspects of the program into each day's lesson. Teachers thought that there were several difficulties and/or changes which needed to be made in the homework phase: homework should be started in class (although this procedure is part of the program recommendation); the homework phase needs to be longer; and they were concerned that pupils might cheat on homework assignments. In spite of these complaints, six teachers said they would use the program if the modifications they suggested were made, and four teachers said that they would use parts of the program.

Reaction of Partnership Teachers to the Project at the Initial Meeting

The six project partnership teachers who brought their evaluation sheets with them to the initial meeting also reported favorable attitudes toward the project. Three teachers said that the mental computation part of the program was very good; two teachers rated the increased pace as very good (although three teachers reported that the strategy was only so-so); four of the six teachers reported that the verbal problem-solving stage of the lesson was very good. According to these teachers, the weakest parts of the program were seatwork, homework, and increased pace (see Table 1).

In general, teachers indicated that the basic training manual was comparable to instructional methods they had been using. Seatwork and the broad and weekly reviews seemed to be newest to teachers. The teachers reported that they were familiar with most (but not all) of the verbal problem-solving strategies, and that they were generally using the strategies, at least on occasion.

The free responses teachers made to questions about the project materials follow the tables in this appendix and allow a detailed examination of teacher concerns and suggestions. As can be seen, the non-user teachers were much more thorough and expansive in their comments than were partnership teachers. This difference is mentioned in the text of the report, where we emphasized that non-user teachers were much more verbal and expressed much more interest and more positive affect about the program during the meeting than did partnership teachers.

In general, Table 4 indicates that both partner and treatment teachers were reasonably supportive of the program, although there are differences in support within both groups. There does not appear to be any overall difference between partner and treatment groups' attitudes toward the program. That is, there is more difference within than between groups. This affective reaction of teachers in general parallels the implementation scores presented earlier, and help to explain why there were no differences in the effects of partner and treatment teachers.

Partnership Teachers' Pre- and Post-Reactions

One interesting question concerns teachers' reactions to the project before they had the opportunity to teach it and after they had used the program. As can be seen in Table 5, the partnership teachers' responses before and after the project varied considerably. Teacher 1 reported basically the same reaction to the program as a whole and to each of the parts before and after the project. In the case of Teacher 2, all three changes in the rating were positive; and hence, the teacher became more supportive of the program. The responses of Teacher 3 are about balanced, but became slightly more negative after using the program. The same pattern is evidenced by Teacher 4. In contrast, the changes for Teacher 5 were all in a positive direction and this teacher became much more supportive of the program. Teacher 6's ratings became more negative over time. Unfortunately, in the post-assessment forms, the rating of verbal problem solving was inadvertently omitted, and comparisions cannot be made for this part of the program. However, as we will see below, the teachers' free responses indicated a continuing interest in and support for verbal problem solving. On the free responses, four of the six partnership teachers indicated that verbal problem solving was the most important part of the program. However, two of the general treatment teachers thought that this was the weakest part. Across all teachers, the only changes which appeared consistently were a tendency for homework to increase in perceived value and for pace to decrease in value.

Initial Reactions of "Non-User" Eighth-Grade Teachers (N = 12) to the Missouri Mathematics Effectiveness Program

Table 1. General Reactions to the Project

	Very Good	Good	So-So	Little Value	No Value
Entire Program	2	10			
Review	5	7			
Development	4	7	1		
Seatwork	4	6	2		
Homework	4	5	3		
Mental Comp.	5	4	2	1	
Increased Pace	3	3	5		1
Verbal Prob.	5	3	4		

Initial Reactions of "Non-User" Eighth-Grade Teachers (N = 12) to the Missouri Mathematics Effectiveness Program

Table 2. Correspondence to Present Teaching

	Great Corresp.	General Overlap	Some Overlap	Little Overlap
Whole Class	5	5	2	
Verbal Prob.	1	3	6	2
Developmt.	7	4	1	
Seatwork	2	2	7	1
Homework	4	5	2	1
Mental Comp.		4	5	3
Weekly Review	1	4	3	4
Daily Review	3	8		1

Initial Reactions of "Non-User" Eighth-Grade Teachers (N = 12) to the Missouri Mathematics Effectiveness Program

Table 3. Verbal Problem Solving

	Familiarity			Usage	
	New	Familiar	Never	Occas.	Freq.
Problems Without Numbers	4	8	5	5	2
Writing Verbal Problems		12	1	7	4
Estimating the Answer		12		6	6
Reading Verbal Problems	2	10	3	5	4
Writing Open Sentence		12		3	9

Initial Reactions of Partnership Eighth-Grade Teachers (N = 6) to the Missouri Mathematics Effectiveness Program

Table 1. Reaction to the Project

	Very Good	Good	So-So	Little Value	No Value
Entire Program	1	5	0	0	0
Review	1	4	1	0	0
Development	1	5	0	0	0
Seatwork	1	3	2	0	0
Homework	1	2	3	0	0
Mental Comp.	3	3	0	0	0
Increased Pace	2	1	3	0	0
Verbal Prob.	4	2	0	0	0

Initial Reactions of Partnership Eighth-Grade Teachers (N = 6) to the Missouri Mathematics Effectiveness Program

Table 2. Reaction to the Project

	Great Corresp.	General Overlap	Some Overlap	Little Overlap
Whole Class	4	1	1	0
Verbal Prob.	2	4	0	0
Development	2	3	0	0
Seatwork	1	1	4	0
Homework	3	2	1	0
Mental Comp.	1	4	1	0
Weekly Review	0	2	4	0
Daily Review	2	3	1	0

Initial Reactions of Partnership Eighth-Grade Teachers
(N = 6) to the Missouri Mathematics
Effectiveness Program

Table 3. Verbal Problem Solving

	Familiarity		Usage		
	New	Familiar	Never	Occas.	Freq.
Problems Without Numbers	1	5	2	2	2
Writing Verbal Problems	0	6	1	2	3
Estimating the Answer	1	5	1	2	3
Reading Verbal Problems	0	6	1	2	3
Writing Open Sentence	0	6	0	0	5

Table 4. Summary of Partnership and Regular Treatment Teachers' Reactions After Participating in the Mathematics Project

	Partner 01	Partner 02	Partner 03	Partner 04	Partner 05	Partner 06	Treatment 07	Treatment 08	Treatment 09	Treatment 10	Treatment 11
I. Reaction to the Various Phases 1=very good, 2=good, 3=so-so 4=of little value, 5=of no value											
Entire Program	2	1	3	3	1	2	2	2	2	2	2
Review	2	1	2	3	1	2	2	1	2	1	2
Development	2	2	2	3	1	3	2	2	3	2	2
Seatwork	3	2	4	2	2	3	2	2	3	2	2
Homework	2	2	2	2	1	3	2	2	2	1	2
Mental Computation	2	1	1	3	1	2	2	4	2	2	2
Pacing	2	3	4	4	1	2	2	5	3	3	3
Average	2.14	1.71	2.57	2.86	1.14	2.43	2	2.57	2.43	1.86	2.14
II. Classroom Behavior After the Project 2=continue the program with minor modifications 4=continue a few parts of program	2	2	4	4	2	4	4	2	4	2	4
III. Best Part of Program	Review & verbal problems	Gave fullness to instruction (a much wider presentation of math emerged)	Problems without numbers	Verbal problems	Review & development of each topic with homework assignments	Development and verbal problems	The consistency of doing review, mental comp etc. each day	$100 Having some order to classroom activities	Verbal problems	Checking homework, review & development	The review & development
IV. Weakest or Most Confusing Part of Program	The use of both seatwork & homework	The increase of pace was almost prohibitive	Trying to keep the time schedule	The time allotments. The posttest was not over the material taught	Trying to get it all worked into a 55-minute class period		Trying to get it all worked into the class period	The minute to minute time table	Lack of flexibility in the time frame	Verbal problems	Verbal problems
V. Was Explanation of Program Clear	Yes. The manual was good	Need more on estimation & it should be linked to mental comp.	Yes, but part is impractical	Yes	Yes, but had trouble with working in the timing in actual practice	Yes	Need a demonstration of a full class period using the times	Ok.	Too many examples. Mail out asking for volunteers should be more comprehensive	Yes	Satisfactory

Table 5*

Partnership Teachers' Reactions to the Mathematics Program Before and After Implementation

	Teacher 1		Teacher 2		Teacher 3		Teacher 4		Teacher 5		Teacher 6	
	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
Entire Program	2	2 (0)	2	1 (+)	2	3 (-)	2	3 (-)	2	1 (+)	1	2 (-)
Review	2	2 (0)	2	1 (+)	3	2 (+)	2	3 (-)	2	1 (+)	1	2 (-)
Development	2	2 (0)	2	2 (0)	2	2 (0)	2	3 (-)	2	1 (+)	1	3 (-)
Seatwork	2	3 (-)	2	2 (0)	1	4 (-)	3	2 (+)	3	2 (+)	2	3 (-)
Homework	2	2 (0)	2	2 (0)	3	(2 (+)	3	2 (+)	3	1 (+)	1	3 (-)
Mental Computation	2	2 (0)	2	1 (+)	1	1 (0)	2	3 (-)	1	1 (0)	1	2 (-)
Pacing	2	2 (0)	3	3 (0)	3	4 (-)	3	4 (-)	1	1 (0)	1	2 (-)

[*] A lower number indicates a more favorable attitude.

Initial Reactions of Eighth-Grade Partnership Teachers

Partnership Teacher

Reaction of Eighth Grade Teachers to the Missouri Mathematics Effectiveness Program

I. Reaction to the project

When responding to these questions, please use the following scale: 1 = very good; 2 = good; 3 = so-so; 4 = of little value; 5 = of no value.

_1_ my reaction to the entire program
_1_ my reaction to the review phase of the lesson
_1_ my reaction to the development stage of the lesson
_2_ my reaction to the seatwork stage of the lesson
_1_ my reaction to the homework stage of the lesson
_1_ my reaction to the use of mental computation problems
_1_ my reaction to the increased pace suggestion
_1_ my reaction to the verbal problem solving material

II. Correspondence to present teaching

Please indicate which aspects of the program are already part of your classroom teaching. When responding please use the following scale:

1 = great correspondence between what I was already doing and the program request.
2 = general overlap between what I was already doing and the program request.
3 = some overlap between what I was already doing and the program request.
4 = little if any overlap between what I was already doing and the program request.

_1_ teaching the class as a whole
_1_ verbal problem solving strategies
_1_ development
_3_ seatwork
_1_ homework
_2_ mental computation
_3_ broad review and weekly review
_2_ daily review

III. Verbal problem solving

How different from your current practice are the five verbal problem solving strategies? When responding please use the following scales:

Familiarity	Usage
1 = new to me	1 = never use this strategy
2 = familiar to me	2 = occasionally use this strategy
	3 = frequently use this strategy

Familiarity	Usage	
2	3	problems without numbers
2	3	writing verbal problems
2	3	estimating the answer
2	3	reading verbal problems
2	3	writing an open sentence

General Feedback from Eighth Grade Teachers about the Missouri Mathematics Effectiveness Program

1. General strengths.

Immediate reinforcement of understanding concepts through seatwork practice, and successful completion of homework. Perhaps a more structured well thought out teacher preparation will result from following the outlined program.

2. General weaknesses.

Program was written for elementary students. In order to be effective the modifications must reflect the difference in maturation as well as program and class organization differences.

3. What changes need to be made to improve the program for secondary school application?

Since fourteen year olds (12-14) should accept more responsibility, homework could be more in depth than for elementary students. Also, immediate feed back is more successful since oral recitation in a group situation is more appropriate and less time consuming.

4. In general, would you use the program (assuming the modifications you suggest in question # 3 are made)?

yes

Partnership Teacher

Reaction of Eighth Grade Teachers to the Missouri Mathematics Effectiveness Program

I. Reaction to the project

When responding to these questions, please use the following scale: 1 = very good; 2 = good; 3 = so-so; 4 = of little value; 5 = of no value.

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2 my reaction to the review phase of the lesson
2 my reaction to the development stage of the lesson
3 my reaction to the seatwork stage of the lesson
3 my reaction to the homework stage of the lesson
2 my reaction to the use of mental computation problems
3 my reaction to the increased pace suggestion
1 my reaction to the verbal problem solving material

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1 teaching the class as a whole
2 verbal problem solving strategies
2 development
3 seatwork
1 homework
2 mental computation
3 broad review and weekly review
2 daily review

III. Verbal problem solving

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2	3	reading verbal problems
2	2	writing an open sentence

General Feedback from Eighth Grade Teachers about the Missouri Mathematics Effectiveness Program

1. General strengths.

Verbal problem solving
Mental computation
Broad, weekly, + daily review.

2. General weaknesses.

What type of evaluation?
Is there enough time per day to accomplish all that needs to be accomplished?

3. What changes need to be made to improve the program for secondary school application?

More than fifteen minute for homework.

Try to incorporate to some extent the seatwork + homework stages in order to have more time for the development stage, the pre-development stage + to allow more time for review + individual help.

4. In general, would you use the program (assuming the modifications you suggest in question # 3 are made)?

Yes

Partnership Teacher

Reaction of Eighth Grade Teachers to the Missouri Mathematics Effectiveness Program

I. Reaction to the project

When responding to these questions, please use the following scale: 1 = very good; 2 = good; 3 = so-so; 4 = of little value; 5 = of no value.

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Familiarity	Usage	
[illegible]	1	problems without numbers
[illegible]	3	writing verbal problems
[illegible]	[illegible]	estimating the answer
2	[illegible]	reading verbal problems
[illegible]	3	writing an open sentence

General Feedback from Eighth Grade Teachers about the Missouri Mathematics Effectiveness Program

1. General strengths.

Constant Reviews + Teachs Repetition

2. General weakness.

2 [illegible] ?

3. What changes need to be made to improve the program for secondary school application?

Most everything is compatable with methods that I use. I would need to lighten up on the time sequence.

I see a difficulty in trying to evaluate students in this program for grades.

4. In general, would you use the program (assuming the modifications you suggest in question # 3 are made)?

Yes.

Partnership Teacher

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2	3	estimating the answer
2	3	reading verbal problems
2	2	writing an open sentence

General Feedback from Eighth Grade Teachers about the Missouri Mathematics Effectiveness Program

1. General strengths.

2. General weaknesses.

3. What changes need to be made to improve the program for secondary school application?

none

4. In general, would you use the program (assuming the modifications you suggest in question # 3 are made)?

Yes

Partnership Teacher

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Familiarity	Usage	
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2	2	writing verbal problems
1	1	estimating the answer
2	1	reading verbal problems
2	3	writing an open sentence

General Feedback from Eighth Grade Teachers about the Missouri Mathematics Effectiveness Program

1. General strengths.

THE REVIEW PHASE IS GOOD. I REVIEW, BUT SELDOM ON AN ORGANIZED BASIS. IN FACT, OFTEN IN MY DESIRE TO TEACH ADDITIONAL MATERIAL, I OMIT REVIEW TIME. THIS PROGRAM HAS CONVINCED ME TO ALTER MY VIEW CONSIDERABLY. THE MENTAL STAGE IS ALSO A WORTHWHILE TOUCH.

2. General weaknesses.

WONDERED WHY THE HOMEWORK ASSIGNMENT COULDN'T BE EXTENDED TO ENCOMPASS THE SEATWORK PORTION OF THE PERIOD. NOT ALWAYS — BUT DEFINITELY A PORTION OF THE TIME. DISADVANTAGES WOULD BE LOSS OF IMMEDIATE FEEDBACK FOR EVERYONE — ALSO — SOME DIFFICULTY WITH THOSE WHO KNOWING THEY WON'T BE GRADED IMMEDIATELY OPT TO TAKE THE REST OF THE DAY OFF & DO IT ALL AT HOME. ADVANTAGES — MORE TIME FOR A ONE TO ONE CONTACT.

I'M A LITTLE RELUCTANT TO JUDGE THE INCREASED PACE SUGGESTION UNTIL I'VE TRIED IT. THE EXPLANATION GIVEN IS SOMETHING I'VE NEVER THOUGHT ABOUT. ~~[illegible]~~ (I'M WILLING TO GIVE IT SOME THOUGHT AND A VALID TRY)

3. What changes need to be made to improve the program for secondary school application?

I WOULD LIKE TO HOLD MY COMMENTS UNTIL I'VE INCORPORATED IT INTO MY TEACHING PROGRAM.

4. In general, would you use the program (assuming the modifications you suggest in question # 3 are made)?

Yes

Reaction of Eighth Grade Teachers to the Missouri Mathematics Effectiveness Program

I. Reaction to the project

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3 my reaction to the review phase of the lesson
2 my reaction to the development stage of the lesson
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3 my reaction to the increased pace suggestion
1 my reaction to the verbal problem solving material

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1 teaching the class as a whole
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1 mental computation
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1 daily review

III. Verbal problem solving

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2	3	reading verbal problems
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General Feedback from Eighth Grade Teachers about the Missouri Mathematics Effectiveness Program

1. General strengths.

2. General weaknesses.

3. What changes need to be made to improve the program for secondary school application?

4. In general, would you use the program (assuming the modifications you suggest in question # 3 are made)?

Initial Reaction of Eighth-Grade
"Non-User" Teachers

Reaction of Eighth Grade Teachers to the Missouri Mathematics Effectiveness Program

I. Reaction to the project

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2 mental computation
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Familiarity
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2 = familiar to me

Usage
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Familiarity	Usage	
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		writing verbal problems
		estimating the answer
		reading verbal problems
	3	writing an open sentence

"Non-user" Responses to the Project

Reaction of Eighth Grade Teachers to the Missouri Mathematics Effectiveness Program

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1 development
3 seatwork
1 homework
___ mental computation
___ broad review and weekly review
2 daily review

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"Non-user" Responses to the Project

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"Non-user" Responses to the Project

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2	2	problems without numbers
2	3	writing verbal problems
2	3	estimating the answer but not formally writing it down.
1	1	reading verbal problems
2	2	writing an open sentence

Reaction of Eighth Grade Teachers to the Missouri Mathematics Effectiveness Program

I. Reaction to the project

When responding to these questions, please use the following scale: 1 = very good; 2 = good; 3 = so-so; 4 = of little value; 5 = of no value.

2 my reaction to the entire program
1 my reaction to the review phase of the lesson
2 my reaction to the development stage of the lesson
2 my reaction to the seatwork stage of the lesson
2 my reaction to the homework stage of the lesson
1 my reaction to the use of mental computation problems
1 my reaction to the increased pace suggestion
2 my reaction to the verbal problem solving material

II. Correspondence to present teaching

Please indicate which aspects of the program are already part of your classroom teaching. When responding please use the following scale:

1 = great correspondence between what I was already doing and the program request.
2 = general overlap between what I was already doing and the program request.
3 = some overlap between what I was already doing and the program request.
4 = little if any overlap between what I was already doing and the program request.

2 teaching the class as a whole
3 verbal problem solving strategies
1 development
1 seatwork
1 homework
2 mental computation
1 broad review and weekly review
1 daily review

III. Verbal problem solving

How different from your current practice are the five verbal problem solving strategies? When responding please use the following scales:

Familiarity	Usage
1 = new to me	1 = never use this strategy
2 = familiar to me	2 = occasionally use this strategy
	3 = frequently use this strategy

Familiarity	Usage	
2	2	problems without numbers
2	2	writing verbal problems
2	3	estimating the answer
3	2	reading verbal problems
2	3	writing an open sentence

"Non-user" Responses to the Project

Reaction of Eighth Grade Teachers to the Missouri Mathematics Effectiveness Program

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1 teaching the class as a whole
2 verbal problem solving strategies
1 development
2 seatwork
2 homework
4 mental computation
4 broad review and weekly review
2 daily review

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How different from your current practice are the five verbal problem solving strategies? When responding please use the following scales:

Familiarity	Usage
1 = new to me	1 = never use this strategy
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Familiarity	Usage	
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2	2	writing verbal problems
2	2	estimating the answer
2	2	reading verbal problems
2	3	writing an open sentence

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1 my reaction to the homework stage of the lesson
1 my reaction to the use of mental computation problems
3 my reaction to the increased pace suggestion
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3 seatwork
2 homework
3 mental computation
2 broad review and weekly review
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2	3	writing verbal problems
2	2	estimating the answer
2	3	reading verbal problems
2	3	writing an open sentence

"Non-user" Responses to the Project

General Feedback from Eighth Grade Teachers about the Missouri Mathematics Effectiveness Program

1. General strengths.

① Problems(verbal) without numbers
② making up verbal problems
③ Writing opensentences for verbal problems
④ mental computation
⑤ momentum
⑥ accountability
⑦ general review

2. General weaknesses.

① Not enough time on review, going over homework
② Time schedule - rigid
③ Checking seat work too vague
④ Estimating answers
⑤ Time consuming on accountability

3. What changes need to be made to improve the program for secondary school application?

① Seat work should be checked but it could be left on homework paper and graded along with the homework

② On estimating - instead - the student should check answer for reasonableness

4. In general, would you use the program (assuming the modifications you suggest in question # 3 are made)?

To a certain extent

General Feedback from Eighth Grade Teachers about
the Missouri Mathematics Effectiveness Program

1. General strengths.

The following are good ideas: problems without numbers, writing verbal problems + writing an open sentence & mental computation.

The idea of maintaining momentum is important.

Review is important but what is done with new students gotten near the end of the year or month.

2. General weaknesses.

There is not enough time for daily review & homework checking. Attendance always takes a minute or two.

There may not be enough practice if seatwork + much of the homework except one day is kept short.

I believe a numerical score should be given to seatwork. Comments are not enough motivation for many students. This does not have to be done every day to motivate students. A random ten problems could be checked for 10 pts.

3. What changes need to be made to improve the program for secondary school application?

Examples could be written on their seatwork paper, and used as a guide in working their problems & gets the students actions in learning what they are to do and how it is to be done.

Seatwork & homework should be put on the same sheet of paper.

4. In general, would you use the program (assuming the modifications you suggest in question # 3 are made)?

No

A junior high math teacher has three or more preparations. This program would require too much time to prepare each day. If there were a textbook written along these lines with seatwork problems + review problems in great enough quantities the program would be feasible.

"Non-user" Responses to the Project

General Feedback from Eighth Grade Teachers about the Missouri Mathematics Effectiveness Program

1. General strengths.

I think that the daily and weekly lessons organization plans are good. The structure provides for feedback and a very organized way of presenting new material as well as continually reviewing past material.

2. General weaknesses.

3. What changes need to be made to improve the program for secondary school application?

4. In general, would you use the program (assuming the modifications you suggest in question # 3 are made)?

I use a structure very similar to the new and would alter my approach to include more verbal problem solving strategies.

"Non-user" Responses to the Project

General Feedback from Eighth Grade Teachers about the Missouri Mathematics Effectiveness Program

1. General strengths.

In my opinion the strengths are the emphasis on review and practice. Many teachers have the good intentions of reviewing but somehow never fit it in. The course booklet out about homework spreading a student's practice out over time was also good. The program would be of great assistance to a teacher who has trouble organizing.

2. General weaknesses.

The greatest weakness for me would be the tight scheduling of time. I am a fairly organized person and in general my classes follow the pattern described but I know from experience that I couldn't always adhere to such rigorous time slots. This is especially true in a secondary school where math class is exactly "x" minutes long and then the students move on. I think variety, even if it just means correcting an assignment later in the class period rather than immediately might be beneficial.

3. What changes need to be made to improve the program for secondary school application?

Less rigorous time schedule.

1) Keep the weekly reviews but not necessarily the same day weekly.

2) I don t understand the difference between working practice problems pg. 5 and controlled practice pg. 6. A lot of examples can be integrated into the active demonstration and I feel the practice problems, controlled practice and seat work are a little repetiti

3) I feel it is important to interact personally on an informal level with students in the 8th grade to hold their interest. This program doesn't allow for that.

4. In general, would you use the program (assuming the modifications you suggest in question # 3 are made)?

I agree with many of the ideas brought out in the program. It is really difficult to express my feelings without having talked to you about it beforehand. My main concern is the fact that you have filled every minute of my time with the students. I don t think this would be a problem in elementary schools because of the large amount of time teachers and students have together. If the program is meant to be as strict as it sounds on paper, I don't think I could use it

General Feedback from Eighth Grade Teachers about the Missouri Mathematics Effectiveness Program

1. General strengths.

The program seems well thought out allowing maximum review time and distributed practice. The seatwork and controlled practice are especially suited for basic and general math classes where practice is necessary under supervision.

The verbal problem solving manual is excellent and should be incorporated into every mathematics program.

2. General weaknesses.

It Seems a bit too regemented as far as time.

Nothing is mentioned about testing and how those reviews fit into the schedule.

In a secondary school, seatwork need not be collected — other methods of accountability can be used.

3. What changes need to be made to improve the program for secondary school application?

Allow more time for the daily review for higher mathematics courses such as algebra and geometry.

Make seatwork be the beginning of the homework assignment with spot checking before class ends to insure understanding. This will reduce the amount of paper-shuffling.

Allow for courses with short units to review at the end of a unit and test rather than the weekly review.

4. In general, would you use the program (assuming the modifications you suggest in question # 3 are made)?

I would use the program in a general sense but I prefer to remain flexible. My present teaching methods already strongly parallel the method outlined but are not as rigorous and seem just as efficient.

General Feedback from Eighth Grade Teachers about the Missouri Mathematics Effectiveness Program

1. General strengths. The program in general is a very sound one. It gives the teacher and the student a sense of organization in the general classroom procedures as well as in the learning and instructional areas.

The review phase, if one is able to implement it properly, could play a major role in increasing mathematical retention and proficiency.

Depending on the precision of the teacher in implementing this program, many behavioral problems could be eliminated.

2. General weaknesses.

The weaknesses lie mainly not in the program itself, but in its implementation. The realities of an actual classroom and school situations are not always ideal.

For instance, there are many factors that can effect the total classroom time period.

There are also many non-teaching chores that teachers must do during the classroom hours which may weaken the programs effectiveness.

There was also no mention of testing.

3. What changes need to be made to improve the program for secondary school application?

The homework and seatwork ideas would need modifications, depending on the subject matter and types of students

4. In general, would you use the program (assuming the modifications you suggest in question # 3 are made)?

I would use this program to some degree.

General Feedback from Eighth Grade Teachers about the Missouri Mathematics Effectiveness Program

1. General strengths. There is time for review. Attempt to help teacher become a better questioner and better manager of time by giving guidelines. I found the specific strategies interesting -- kind of question asked. teacher do explaining during the developmental part of lesson. Idea of mental computation good

2. General weaknesses. Reading through this I got the feeling that this could be seen as a regimented approach. I think sometimes an entire day might be a developmental day. Or perhaps approaching math lesson in a less serious vein
25% of grade be homework seems to be unworkable since cheating/copying is frequently possible. It would seem that it would be possible to pass if a student fails all other work but does homework using this system.
The time to go over homework seems too short!

3. What changes need to be made to improve the program for secondary school application?

Change criteria for HW as 25% of grade
Allow more time for HW checking and organizational
matter. Seat work perhaps could be dropped
or reduced -- instead use more controlled
practice when dealing with slower pupils or
poorly disciplined students. Have students check
their own papers during HW.
Change tone of approach and do not make
this mandatory.
Isn't there any room for flexibility or creative
ideas! field trips, computer work, calculator work,
probability, experiments, or active measurements, day of
math games Allow more flexibility or time for different pacing

4. In general, would you use the program (assuming the modifications you suggest in question # 3 are made)?

I would probably only use part of it at first
since it would require quite a lot of change
and new work. Perhaps if 2 or more teachers
I'm not sure I would want to do all that
reviewing. The situation now has some built in
and I'm not sure I would want to add more
since it would mean dropping some
The pace! Why speed up to cover more
material if they don't understand what they had
before -- lag in developmental lesson. I think they
would do better & be more confident if they were not
rushed along

General Feedback from Eighth Grade Teachers about the Missouri Mathematics Effectiveness Program

1. General strengths.

(1) Presents a structured routine to follow.

(2) Allows for good student participation.

(3) Integrates problem solving, mental computation with daily work.

(4) Incorporates a good system of providing daily review for reinforcement.

2. General weaknesses.

(1) method of scoring homework:

(a) students often can not read other students papers (e.g. wasted time)

(b) recording a grade for each assignment results in a large amount of time to be spent averaging grades

(c) When a student does not do well on an assignment, anxiety results because of scoring for correctness.

(2)

3. What changes need to be made to improve the program for secondary school application?

4. In general, would you use the program (assuming the modifications you suggest in question # 3 are made)?

Yes, I would consider using the program.

General Feedback from Eighth Grade Teachers about the Missouri Mathematics Effectiveness Program

1. General strengths.

- Weekly reviews and monthly reviews are a very good idea — much better than reviewing just before a test.

- Looking at Thursday's assignment results in order to structure Monday's review.

- The program is well-structured. In an ideal classroom, it would be easy to implement.

-- The suggestions for teaching word problems are *very* helpful to me.

2. General weaknesses.

- I would probably become bored after awhile from doing the same things in the same order every day. Perhaps the students would also.

- I would not have enough time to check everyone's seatwork before homework is assigned due to large class sizes.

- Grading homework seems to be a routine. Students may get into the *habit* of cheating to help themselves or their friends. The teacher needs to check some homework other than Thursdays.

- I would spend too much time recording homework grades at home if I did that every night. Some time needs to be spent on recording during class (at *least* once in a while).

3. What changes need to be made to improve the program for secondary school application?

- What do you suggest for tests? How often? How many problems? Chapter tests and/or cumulative tests?
- Quizzes?
- More homework problems than suggested.
- Our classes are only 40 minutes. Could certain parts of the lesson be shortened?
- Change the homework grading procedure so that the teacher does not have so much recording to do.

4. In general, would you use the program (assuming the modifications you suggest in question # 3 are made)?

Yes, I would use the program in at least one or two of my classes. I'm sure it would be successful, but I am a little concerned about following the same routine every day.

I am going to start using the verbal problem suggestions.

General Feedback from Eighth Grade Teachers about the Missouri Mathematics Effectiveness Program

1. General strengths.

Verbal problem solving Manual is excellent. [illegible] ideas [illegible] lesson is [illegible] [illegible] is very [illegible] the program is well-structured. very nicely done.

2. General weaknesses.

1) The time limit suggested is inconsistent with the class schedule (40 minutes)

2) Seatwork assignment should be a specific assignment (not "work as far as you can get"). In a class of 30 kids, it is almost impossible to grade each kid's given assignment (subjectively).

3) Homework assignment should be on the board [illegible]

4) Teachers should not be restricted [illegible] 18 [illegible]

5) I don't like the idea of Thurs' assignment resulting in structuring Mon. day's review - why let the kids be confused for 3 days? Why not have a quiz Friday and correct misconceptions then

3. What changes need to be made to improve the program for secondary school application?

quizzes? How about a quiz Friday – this could structure Monday's review
Tests?
enrichment?
change in routine
What about the 40 min. classes? the travelling teacher?
Homework should be long enough to cover at least 3 of each type of problem –
How about word problems with every quiz or homework assignment

4. In general, would you use the program (assuming the modifications you suggest in question # 3 are made)?

yes

"Non-user" Responses to the Project

General Feedback from Eighth Grade Teachers about the Missouri Mathematics Effectiveness Program

1. General strengths.

1 Use of review
2. Emphasis on momentum
   a) During controlled practice teacher does not allow students to dwell on a problem for more than a minute p6
   b) During the seatwork time students are started properly and monitored to keep them working
3. Students are accountable for all their work

2. General weaknesses.

1. Seatwork collection at the end of the period
takes away all work that could help the
student do his homework. Parents cannot
see what they did in class and may not
be able to help them. Allowing students
to take seatwork gives them examples.

3. What changes need to be made to improve the program for secondary school application?

In general the ideas are good. I felt pressured when I considered my classes of over 30. There is too much checking of papers and recording of scores to keep the momentum up. I would have to make the students more accountable to themselves without checking on them regularly.

4. In general, would you use the program (assuming the modifications you suggest in question # 3 are made)? yes

Appendix 6

Interview Responses of All
Partnership Teachers and Some Treatment Teachers
to the Mathematics Project[1]

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[1] These date are drawn from interviews conducted by Dr. Dee Ann Spencer-Hall after the project was terminated. For a discussion of these data, see pages 37 to 40 in the text. Note that in order to mask the identity of individual teachers, all teachers are referred to as "she" in interview descriptions although about half of the teachers were males. Much of the rich description is lost because statements describing a unique practice or setting were omitted again to mask the identity of respondents.

Teacher 01 (Partnership Interview)

Math Program. Teacher 1 didn't think she had participated much in the project. She was "too old to learn new ways." Her main criticism was that it was too confining and inflexible. She didn't think the text allowed 20 minutes of development and preferred the old Laidlaw book because it had more "depth."

Researchers. I can only describe Teacher 1's reaction to participation in the program as very resentful. She felt that she was forced from the beginning, that it was a "hurry-up deal," and that the principal coerced teachers to participate at the last minute. She compared it to an enjoyable science project she participated in a few years ago. The science program had more meetings, the leader was relaxed about it and teachers participated in the planning. Teacher 1 felt that in the present study the researchers said it *must* be done this way, and that because there was only one brief meeting, teachers had no real input. She was upset about the lack of feedback and interaction with the observers. Her students also would have liked feedback.

Teacher 02 (Partnership Interview)

Math Program. Teacher 2 was very positive about the program. She tried to follow the schedule exactly. Her implementation went slowly at first, but gradually smoothed out. She felt the program was so effective that she will use it from now on. She had some good specific suggestions for change. She thought there was a need to develop the area of estimating, to "beef it up." It took her too long to lay the groundwork for estimation to the point where students could do it. She combined the mental estimation and the word problems and said she would suggest that the review also be combined with mental estimations.

She felt a weakness was that it took so long (two weeks) to smooth out the routine. In general, it was harder to use the program with lower classes (i.e., slower classes require more time, both for development and for discipline, which disrupts the routine). However, this teacher stated that by going more slowly in all classes and also by expanding into more areas, that the slower classes could feel they were keeping up for a change.

Teacher 2 did mention that the mental computation was particularly good and that her students loved it. In retrospect, she feels that she didn't go as far in the book as she would have normally, but because she did so much development in more areas, in the long run, students will have covered as much material as they normally would have by the end of the year.

In talking to other teachers, she said there were mixed reactions to participating in the project, i.e., some liked it, some didn't, and some were in the middle. She thought these reactions resulted from the fact that everyone wants to teach things his/her own way.

Researchers. Teacher 2 had little negative reaction, but said she was a bit frustrated at first because she was reluctant to criticize the program because she had never used it before. Teacher 2 decided she would try the program first and then discuss her reaction (as in the interview). She thought it would improve projects where teachers and professors work together if professors got back into the classroom (to teach). She felt professors need more realistic input rather than learning things second-hand from teachers.

She was worried that her students didn't understand why observers were in the classroom. Students were bothered, as was Teacher 2. She was also curious about researchers' conclusions and was not aware that she would be getting any feedback on the results.

Teacher 03 (Partnership Interview)*

Math Program. Teacher 3 had a mixed reaction to the program. She was using the techniques in the program already, but thought the schedule was too rigid. She was primarily critical of not grading homework. Teacher 3 spent a lot of time giving me a detailed explanation about why and how she graded homework. She was positive about the word problems and the idea of reading them without numbers. This teacher will change to the extent that she will incorporate this style of teaching word problems.

Researchers. Teacher 3 had no problem with the early meetings, but felt self-conscious when she was observed, because she didn't think she'd really done her part. Her students were curious about the observers and explained to the students that they were part of a large number of participants and would just be statistics, not individuals. Teacher 3 predicted that her students would do no better on the post-test than on the pre-test; therefore, she hoped it was "worth all the money they spent." In summary, she said she didn't mind participating, but on the other hand, it was a bother.

*NOTE: This teacher was omitted from the sample because she failed to implement the program. The correspondence between verbal and actual behavior is not always high! See also the description of the two observers of this teacher in Appendix 8.

Teacher 04 (Partnership Interview)

Math Program. Teacher 4 thought the math program was exactly what she did in her classes already. In fact, she added that she wouldn't have agreed to participate if the methods hadn't been so similar to what she did.

She said the strong point was in the area of problem solving and mental computation. The weak point was the idea of having both a daily assignment and homework--this puts too much paperwork on the teacher. She had no suggestions for change.

Researchers. Teacher 4 felt the meetings last fall were quite beneficial and had no other comments.

Teacher 05 (Partnership Interview)

Math Program. Teacher 5 was enthusiastic about the program and explained that she asked to participate, despite the fact that she had only one eighth-grade class and three ninth-grade classes. This teacher liked the program because it went along well with what she has always done; therefore, she didn't change much. She tried to follow the schedule closely, but she sometimes ran out of time.

Teacher 5 felt the main strength of the program was that it made her teach. Teachers have to be prepared. She also liked the idea of being told to push the students, and found that she accomplished more than in past years.

According to this teacher, there were no weaknesses in the program, except that there was not enough time to work everything in. She questioned the use of a different pre- and post-test, and thought there would be trouble correlating the two.

Researchers. Teacher 5 had a positive reaction to the meetings, but said that she thought some teachers thought the project "was a big joke," or that it "would be too much work." She said others thought, "Oh, it's easy, we'll just pretend like we're doing it."

She wondered how much data the observers obtained, and stated that they needed to have come more frequently. In general, she thought there would have been more negative reaction to the project if the small group hadn't met first and then gone out and talked to other teachers in a positive way.

Teacher 06 (Partnership Interview)

Math Program. All of Teacher 6's comments about the math program were positive, because the program was exactly what he had already done all these years anyway. The major strength was that the program was highly structured. Teacher 6 was a very structured teacher, and the program merely reinforced her attitudes toward a structured approach. Another strength was the story problems. However, she did comment that the story problems available to her were so simple that she changed them. Teacher 6 didn't feel that she integrated the story problems well until the last two weeks. She suggested no change in the program. She did often incorporate the seatwork and homework portions into the same time segment.

Researchers. Teacher 6 thought the fall meetings with the researchers were quite good, that there was a true exchange between the researchers and teachers. She suggested, however, that two meetings instead of one be held with the partnership teachers.

Teacher C7 (Treatment Interview)

Math Program. This teacher was very positive about the project, and it was what she had always done. Its strength, she felt, was in the structure. The program was logical and what all good teachers do. She thought the general structure could be applied to any course.

The program's major weakness was that there was too much time spent daily on word problems. She suggested, in terms of changing the program, that more inservice training be used and that the program be used in education classes.

Researchers. Teacher 7 was positive about the meetings and her participation. She did, however discuss the exhausting nature of teaching, and said that people in research should come back to teach for about nine weeks so they could remember what it's really like.

Teacher 08 (Treatment Interview)

Math Program. Teacher 8's primary reaction to the program was that it was "too rigid and inflexible." She felt it was impossible to follow it closely because of the unpredictable nature of daily life in school. She pointed out that while we were talking, her class had been interrupted three times, twice to call groups of students out to have their teeth checked and once to announce an assembly later that day (which would cancel one of her math classes). Because of these interruptions and the fact that eighth-grade students don't behave in predictable ways, she felt that teaching couldn't be done in such a routinized way. Teacher 8 suggested that the program was good as a blueprint for beginning teachers. In general, she had been using the program for nineteen years; therefore it was not useful.

Researchers. Teacher 8 told me she brought up her concern about the program's "inflexibility" at the fall meeting, but was "put down," made to feel out of line, and got no support from the other teachers.*

She was upset that there was no teacher or student interaction with the observers, particularly because the students didn't understand why observers were there.

*NOTE: This "occurred" at the training meeting, not at the partnership meeting.

Teacher 09 (Treatment Interview)

Math Program. She started the program with the lower half of the eighth-grade in two sections, but many of the students had been in special education or remedial classes previously. One section couldn't keep up because they needed too much drill. So she eventually used the program in one class--a low eighth-grade class which used a seventh-grade book.

Teacher 9 thought the program was not too much different from the way she had taught in the past; therefore, she didn't change very much. She took some time before the students finally got used to the routine. The program was difficult to implement, however, because it was developed for one-hour sessions and she had only forty-five-minute periods.

The strength of the program was that it was a good approach to math, outlined in a way which incorporates constant review. "You keep drilling, checking, the regular old grind without it seeming like that."

She felt the weakness was that teachers have to have written problems every day--impossible. Teacher 9 thought verbalizing was difficult, but added that maybe she didn't understand what she should have done. She also felt that the twenty minutes for development may be too much for some classes, but not enough for others.

Teacher 9 suggested putting some variation into scheduling. However, in general, she is still following the outline, but is adding to it.

Appendix 7

Descriptions of Effects of Program Planning Practices of Treatment Teachers

The data presented here are drawn from conversations that the two classroom observers had with teachers after they had begun to use the program. In order to mask the identity of individual teachers, all teachers are referred to as "she" even though about half the teachers were male. The following questions were raised in the conversation with teachers:

1. On the average, how long do you spend planning for each preparation?
2. Did you spend more or less time planning for the project classes?
   A lot more, More, Same, Less, A lot less.
3. Was the planning any different?
   _____ Yes   _____ No
   If Yes, how?

01 1. Approximately 15 minutes a day.
She also has to make out weekly lesson plans so this adds additional time to this amount.
2. Less, because one project class is a better class than others.
3. Not really different, because the MMEP program fits exactly what the new Holt book asks them to do, so just in planning normal lessons she's also doing what our program asks for.

02 1. Approximately 15 minutes. It varies quite a bit.
2. Definitely more, she had to write out the verbal problems to be used and plan the mental comp lesson, activity and/or the problems, so it took longer.
3. The planning was definitely different, mainly in the two ways mentioned above (i.e., planning the verbal problem solving and mental comp).

03 1. Five minutes a day. She writes out lesson plans each Friday for the next week, so little daily planning is needed.

2. Same. All classes take about the same amount of preparation, however, first hour is a much better class than second and second often gets behind and doesn't get everything done that she wanted.

3. Not really different.

04 1. 5-10 minutes. She's done it for so long that it doesn't take too long.

2. Pretty much the same for all.

3. No (she laughed a little guiltily, but didn't give any apology, reason, or comment, just the "No.")

05 1. 20-30 minutes a day because she's done it for so long, at first (or when she gets new books) she spent an hour a day planning for each hour of class time.

2. More. It took longer because she had to decide on the mental comp and seatwork problems and work them into the lesson.

3. Different in that she was working in the mental comp and seatwork problems. It was also different because she put in the verbal problems.

06 1. 5-10 minutes a day. She plans 6 weeks at a time and mimeographs assignment sheets for the students and, therefore, she does a lot of planning then; for daily planning only 5-10 minutes.

2. At the beginning of the project she did more (comparing to see how what she was doing fit with the MMEP program). It all fit except for the verbal problem solving so she kept on with her usual routine, except that she tried to work in verbal problems, but she said that she didn't get them started until December.

07 1. 20-30 minutes a day. She spends one hour a day planning.

2. Same.

3. The only difference was that she included a workbook for the verbal problems and for the mental comp problems. She planned problems from it for the project classes.

08 1. 10-15 minutes a day.

2. Same.

3. Not really any different but it did make her more conscious of time (i.e., not to spend too much time on some things so she'd have time to get all activities worked in).

09 1. 30 minutes a day. She likes to plan pretty thoroughly.

2. A little more.

3. The only real difference was that she included daily verbal problem solving; whereas before she had just done verbal problem solving as it came up in the regular lesson.

10 1. Approximately 20 minutes.

2. Definitely more.

3. It was different because she was more thorough with the planning for one project class (i.e., trying to work everything in and getting the times right). In her other classes she just plans one or two activities and then waits to see what develops during the hour, to determine whether there's time for other activities or what needs to be done.

11 1. Only 5 minutes now because it's the same book that she's used for years. The first year she spent about 30 minutes a day in per preparation.

2. Less. She spends 10 or so minutes a day on the other, but again, it's because she has taught from that book for so long.

3. The planning was different in that she worked in the verbal problem solving and the mental comp activities.

Appendix 8

Observers' Brief Sketches of Teachers

Comments About Teachers: Observer I

01 Wasn't very enthusiastic in implementing the program. Most of the quality ratings were about 3. Seemed to spend a lot of time on seatwork or worksheet review classes. About a 3 on implementation. All 3 observations were classified as review. No severe discipline problems, though the quality of student attention was mediocre.

Seemed organized.

Mediocre treatment teacher.

02 Enthusiastic about the program and an enthusiastic teacher. Overall quality ratings were averaging 4-5, 4 for implementation quality scores. Students were contributing to each lesson, good classroom atmosphere. Organized. Quality of student attention high. Excellent treatment teacher.

03 Abrasive in dealings with students. Had severe discipline problems. Was hardly implementing the treatment--degree of implementation a 1. Wasted class time. Quality rating--2's. Not a good teacher by any stretch of the imagination. Terrible treatment teacher.

04 Enthusiastic teacher. Implemented the program well--average 4. Quality high-4. Time on components was slightly off--that kept her from being a 5. Pace was better than most teachers--kept things moving. She was creative in using mental comp. Excellent treatment teacher.

05 Was initially enthusiastic but seemed to have difficulty getting act together. Implementation scores were 4, 3, 2, 3, 2, 1 over my observations. Quality was about 3+ down the line. Spent too much time going over homework and on mental computation. Hardly any time on seatwork. I would classify her as a good teacher but only a fair-poor treatment teacher. Occasionally she'd have to settle the kids down, but they were fairly attentive. Hardly any time on VP.

06 Slow to use the program. In fact, I doubt she changed her routine at all. Spent an enormous % of time going over homework. No time on MC or VP. Little time on DE. I would rate her a 2- on implementation and 3 on quality. She had discipline problems; her rapport was not very high. A poor treatment teacher.

07 Fluctuated quite a bit in implementation. Of the 7 visits, 4 were 1's; 1 was 3; 2 were 4's. Some days she was with it, other days she was completely off track. Quality was a consistent 3. Expressed some difficulty and dislike for having such a rigid, full schedule (time-wise), but said she was giving it a try. A fair to poor treatment teacher, if we take an average.

08 Had good rapport with students. No discipline problem at all. Didn't implement the program all that well--about 2+. Quality was a 3 average. Of 7 visits she did MC once & VP once. She seemed to stay pretty much with what she was doing before the treatment.

09 Fluctuated quite a bit in quality of implementation, going in order from 4, 4, 1, 1, 2. I think 2 of my later visits caught her on unusual days and she did what was best, given the circumstances of the situation. She averaged a 4 on quality and I think overall was a better treatment teacher than her scores indicate.

10 Had serious discipline problems most of the semester and I suspect was not very effective as a teacher. She had no real rapport and was constantly correcting individuals. She averaged a 2 in both overall quality and degree of implementation. Toward the end she seemed to be making some progress toward treatment implementation.

11 Liked the program and made attempt to implement it, though probably not as much as she thought she did. About a 3 on quality and a 2+ on implementation. Had some discipline problems. Too much time on

homework, not much on MC or VP. Average treatment teacher, or slightly below.

12 A good control teacher. Quality about a 4 average. She would have made a good treatment teacher because she averaged a 3 on implementation. Had good rapport with her students and used more than just the textbook in her presentations.

13 An excellent control teacher. Quality was between 4-5. Her implementation was a 2 because she spent her time on 2 components. One day it was DE & PS, another HW & DE, another VP & PS. She had great rapport with her students and used more than just the textbook in her presentations.

14 Had little rapport with her students. When the students disrupted class she threatened them with a graded assignment due at the end of class. About 70% of class time was seatwork. She scored a 2+ on quality and 1+ on implementation. Very little time on development. A poor teacher overall.

15 Averaged a 2 in quality and a 2 in implementation. Had good class control, fairly good rapport, but tended to be boring. The % of time on task was a little lower than others because of this, I think. Students weren't disruptive, just not engaged. Pretty much a homework, development, seatwork teacher.

16 Organized, but had some discipline problems. Was rather monotone in presentation. Averaged a 3 on quality and a 2½ on implementation. Had RE in most (5 of 8) of her lessons, and a substantial development time.

17 Had serious discipline problems. No class control. Students did little work, and roamed the class in control. Usual practice was some HW, then short DE followed by PS for rest of period. A 1 on implementation

and a 1+ on quality. Whatever was learned in this class was done independent of the teacher.

18 Ruled with an iron hand. Students were attentive out of fear more than interest. She took pride in calling herself a traditional teacher. Was organized. Wasn't much on alternative approaches. Rated a 3 on quality and a 2- in implementation.

19 A fine teacher. Good rapport. Had students on task all the time. Had a faster pace than most teachers. Averaged 3 on implementation and 4 on quality. Healthy amount of time on development.

Comments About Teachers: Observer II

01 Seemed unenthusiastic about the project and didn't seem to try to implement the program very much. On 2 of my 5 visits she had mental comp but on both occasions she said "We better do this today," giving me the clear feeling that it was because of my being there.

Three times her only lesson was verbal problem solving (these were weeks apart and problems that she found scattered throughout their books or from another book that she passed out (again, like an activity for my benefit).

02 An excellent teacher who really seemed to try to implement the treatment. She tended to spend too long going over homework, reviewing and doing mental comp and often had only a short amount of time left for development and/or practice seatwork, but it was clear that she was really trying to do it all right.

Mental comp and verbal problem-solving activities were very good.

A good treatment teacher.

03 Would have been the perfect control teacher. The students did crafts 5 weeks during the program. On these days there was no lesson, the students sat, visited, and worked on crafts.

Never did mental comp or anything that remotely resembled the treatment.

Was constantly screaming at the students.

04 A very good teacher who seemed to really try to implement the treatment. She did most parts each day. Times were off some but in general she did well. She was always well in control of the class.

05 Seemed to really try (but often without real success) to implement the treatment. She spent way too long going over homework each day, even though she tried to hurry it along.

About halfway through the programs, she started writing each week's schedule on the back blackboard. She'd list certain mental comp, verbal problem solving and practice seatwork exercises planned for each day. However, usually she'd run short on time and have to announce that they were going to have to skip at least one of these activities that day. Therefore, though she tried, she really didn't implement the treatment very well. However, she did seem to really care about the program and about doing her part.

06 Didn't seem to try very hard to implement the treatment. She never had mental comp and rarely had verbal problem solving. Her daily routine was: go over homework, have a lesson, then seatwork.

I don't feel that she changed much because of the treatment.

She would have made a better control teacher.

07 Didn't seem to care about trying to do the treatment. Whenever I would ask to schedule a visit she wouldn't even note the date or hour. She'd just say, "You won't bother me. I'm not going to change anything."

She rarely did mental comp or verbal problem solving.

She told me that the reason she volunteered was because when she read about the program it sounded just like her usual. Later we added to the program and changed the time limits and she said it was too confining.

I don't think she changed much of anything because of the program.

Even then her implementation scores were usually fairly high.

08 Sometimes would really try to implement the treatment (once she had everything perfect). Other times she wouldn't try at all. She rarely had mental comp or verbal problem solving and her lessons were too long.

She's a very good, interesting and clear teacher and I can't imagine any student not liking her or her class. I feel that if she had tried on all occasions to implement the treatment, she could have been a big help in the study.

09 A very good teacher who really seemed to try to implement the treatment. However, she rarely had the mental comp.

She usually started her first hour class about 10 minutes early to have more time to work things in (15 minutes was allotted to homeroom and as soon as business was completed she began the lesson).

A good treatment teacher.

10 Speaks in a dull monotone and then gets after the students for not paying attention. Every time I observed her class she told me about the low-ability students, the high absentee rate and tardiness rate and how they hadn't learned to behave, but how hard she was trying to straighten them out and how much progress she was making.

She never seems to try too hard to implement the treatment, but in each class she blamed it on the students. In my interview with her she told me how thoroughly she was planning so as to work in all of the treatment.

She would have made a better control teacher.

11 Never really seemed to try to do the treatment. She would spend about 20 minutes going over homework and she rarely had mental comp. Halfway through the program she asked me about the logs. She had been counting the questions the students asked about homework as review and counting something else as mental comp. I tried to explain it all to her again, but I would guess that at least up to that point she was checking about everything each day without really doing anything too differently from before the program began.

She would have made a better control teacher.

12 A very good teacher. She teaches in a very interesting, clear and enthusiastic manner. I got the feeling that the students really liked her and felt like she really cared about them.

She was a control teacher. Her daily routine was to go over homework, have a 20 minute or so lesson, then have seatwork.

She would have been a good treatment teacher and from getting to know her I feel that she would really have tried very conscientiously to implement the treatment.

13 An excellent teacher. She is very enthusiastic and I can't imagine any student not liking her or her class.

She was a control teacher. Her daily routine was a short review over the homework, check the homework, a lesson and then seatwork. She had excellent discipline up to the seatwork, but there she let them be a little too free, in my opinion.

She would have made a good treatment teacher.

14 Always had the students do seatwork most of the hour when I was there. Usually it was strictly review seatwork. Sometimes it was preceded by a very short lesson. My presence seemed to make her nervous despite the fact that she was only a control teacher and not being asked to do anything special. She just always planned seatwork when I was to be there.

However, that could have always been her routine, but from the reactions of the student, I question that. She had discipline problems often, trying to keep the students working.

She made a good control teacher.

15 A very cooperative, friendly person, but as a teacher she speaks in a dull, uninteresting tone which would make it hard for students to concentrate on the lesson.

She was a control teacher. Daily routine was to go over homework, have a 20 minute or so lesson and have seatwork (which she actively supervised very well).

16 Stood at the front of the room and talked to herself all hour. She could teach a 20-30 minute lesson on an hourly review with no one paying any attention to her and as long as students were quiet she didn't appear to notice. She would turn around and write on the board for several minutes at a time while paper wads flew everywhere.

She was a control teacher. Her daily routine consisted of: Going over the homework, a 20-30 minute lesson, and seatwork.

She made a good control teacher.

17 A non-participating control teacher who almost always just had the students work on seatwork. Sometimes it was purely review and other times she gave a very short lesson or had one of the students read aloud from the book the development. Even in checking homework she often had a student read aloud the answers from the book.

She seemed very unenthusiastic about teaching but she was friendly and welcomed my visits. I got the feeling that seatwork was the general daily routine whether or not I was there. She didn't try to have any discipline in the class.

A good control teacher.

18 Seemed to resent the project at first. She got one class out of the study and tried to get the other out.

However, after we got going I think she felt all right about it. She got so she'd visit with me before, after, and even during class (seatwork or boardwork).

She was a control teacher. Routine was varied but included a lot of practice boardwork.

A good control teacher.

19 An excellent teacher. She was always in complete control of the class with no discipline or other problems. She was very well organized, clear, enthusiastic, and interesting.

Appendix 9

Observational Forms and Teacher Logs Used in the Eighth-Grade Mathematics Project

Coding Instructions

Junior High Mathematics Study

On the accompanying coding form indicate in the first column the part of the treatment the teacher is engaged in using the following codes: RE for review, HW for collecting, grading and dealing with homework, MC for mental computation exercises, DE for development, PS for practice seatwork, PB for practice boardwork, VP for verbal problem solving or TR for transition. (See Description of Lesson Components for a description of each of these.) Then place the start time of the activity in the next column.

At the end of each part, rate the quality displayed by the teacher during that activity according to the scales described in the Quality Rating Scales for Lesson Component Parts. Whenever the teacher stops the particular activity (even if only for a few minutes), code the start time of the next activity and assess the quality of the previous activity.

Before class, randomly select 6 students as follows: 2 high achievers, 2 average, and 2 low achievers from those suggested by the teacher for each category. Place their student numbers in the column marked "List the 6." (1-4 means first row, fourth chair.) Then each time one of these students is called on to answer a question, put a "/" beside the student number. If the student then answers correctly, make the mark into an "X." Likewise, whenever one of these 6 asks an academic question, place an "A" in the appropriate column. If he asks a nonacademic question, put an "N." A seating chart for each class is necessary in order to later link student numbers with student names.

Every 10 minutes make a complete sweep of the room, coding each student's behavior as either "on-task" or "off-task." Put a "+" for "on-task" or a "-" for "off-task." If in doubt, continue to observe the student until he either shows signs of being "on-task" or of being "off-task." Then proceed to the next student and code his behavior.

Every 10 minutes relist the 6 selected student numbers along side the time-on-task student numbers for that 10-minute interval and continue to mark /, X, A or N. Also continue coding time-on-task. Thus, there will be a time-on-task measure for each student for each 10-minute interval of classtime, and a record of the number of times each selected student was called on, etc., during the interval.

At the end of the development phase of the class period, fill in the Development codes at the far right of the form. (See Description of High Inference Scales.)

At the end of the class period, place the word "END" in the first column and the concluding time. Fill in the Summary Codes at the far right of the form (again refer to the Description of High Inference Scales for clarification), and list the major topic(s) of the class period.

Fill in the flip side (Check List) immediately after the particular phase of the class period indicated. Place a check mark beside each statement that holds true.

DATE ____________ SCHOOL ____________ TEACHER ____________

TIME ________ OBSERVER ____________ OBSERVATION # ______ PARTNER, TREATMENT, CONTROL

PART OF TREATMENT	START TIME	Student #	Time-on-Task	Student #	Time-on-Task	Student #	Time-on-Task	Student #	Time-on-Task	Student #	Time-on-Task	Student #	Time-on-Task	Student #	Time-on-Task	Student #	Time-on-Task	LIST THE 6	CODE /X	SELF INITIATED QUESTIONS A or N	QUALITY

DEVELOPMENT CODES

CLARITY
1__2__3__4__5__

ENTHUSIASM
1__2__3__4__5__

MANAGERIAL
1__2__3__4__5__

ACCOMPLISHMENT
1__2__3__4__5__

INTERACTION
1__2__3__4__5__

QUALITY
1__2__3__4__5__

SUMMARY CODES

CLARITY
1__2__3__4__5__

ENTHUSIASM
1__2__3__4__5__

MANAGERIAL
1__2__3__4__5__

ACCOMPLISHMENT
1__2__3__4__5__

INTERACTION
1__2__3__4__5__

QUALITY
1__2__3__4__5__

IMPLEMENTATION
1__2__3__4__5__

CHECK LIST

Junior High Mathematics Study

_____ Monday ? Date _____________ Teacher ____________________

Observer _______________ Observation # ___________

Place a check mark (✓) in the blank next to all statements that hold true

INTRODUCTORY PHASE

1. _____ A brief review of previous work was conducted in a meaningful context.
2. _____ 2-5 minutes was spent checking homework.
3. _____ 1-5 minutes was spent on mental computation.
4. _____ There was a slow transition to the main part of the lesson.

DEVELOPMENT

5. _____ Development moved too slow.
6. _____ Development moved too fast.
7. _____ Teacher paced development with progress of students.
8. _____ The students were held accountable for controlled practice during the development phase.
9. _____ The teacher spent 15-21 minutes developing the mathematics in the lesson (disregarding verbal problem solving).

SEATWORK

10. _____ Teacher spent 10-20 minutes on seatwork.
11. _____ The teacher insured that students were actively engaged in seatwork during the first 1½ minutes.
12. _____ The teacher was available to provide immediate help and actively supervised student seatwork.
13. _____ Students were held accountable for the seatwork at the end of the seatwork phase.
14. _____ Seatwork directions took longer than one minute.
15. _____ The teacher assigned homework.

VERBAL PROBLEM SOLVING

16. _____ The teacher spent approximately 10 minutes on verbal problem solving.
17. _____ The teacher spent the majority of the period on verbal problem solving.
18. The following verbal problem solving techniques were used.
    _____ Problems without numbers.
    _____ Students were asked to create and write verbal problems.
    _____ Students estimated the answers to verbal problems.
    _____ Attention was given to the reading of verbal problems.
    _____ Students were asked to write open sentences.

Description of Lesson Components to be Coded

Junior High Mathematics Study

The following is a description of each of the phases of the class period which will be coded. Whenever the teacher's behavior fits the particular description, code it as follows:

Development (DE)

The development portion of the class period is that part of the lesson devoted to establishing comprehension of skills and concepts. Development relates almost exclusively to work with new ideas, concepts and skills. Activities used during this phase often include teacher explanations and demonstrations, and may include the use of manipulative materials, concrete examples, making comparisons and searching for patterns, class discussions, group work, and the use of audio-visual materials.

Often during some part of development, the teacher will pose oral questions to students to assess their comprehension of the topic at hand. The teacher may also use controlled practice during this phase. That is, one or two problems are given at a time and then immediate feedback is given on the correctness of responses. This is usually done while the teacher is trying to identify and correct student misunderstandings. A brief summary of the prerequisite ideas and skills necessary to do or understand the topic of the day is considered part of development. Note that its focus is to facilitate the development portion of the lesson and not simply to refresh past skills and concepts.

Practice Seatwork (PS)

Seatwork refers to practice work which students complete individually at their desks. Seatwork refers to written work only and does not include oral practice.

Practice Boardwork (PB)

Boardwork refers to practice work that students complete individually at the board. Like seatwork, this work relates primarily to work with new ideas, concepts, skills and objectives which were presented in the development phase of that day's class period.

Review (RE)

Review refers to work on old material. It deals with concepts which the students have had prior to the particular day in question (whether it be material from the previous day, or from much earlier). Use of games, puzzles, worksheets, etc., which are used to review skills and ideas fall into this category.

Mental Computation (MC)

Mental computation is computation that is done without the aid of pencil and paper (or minicalculator). The processing is done mentally. The problems which the students are asked to solve mentally may relate either to new or to old ideas.

Homework (HW)

This phase includes all activities involving homework. Thus, it includes the teacher reading off the correct answers to the homework, the in-class grading of homework, the showing of solutions to homework problems, and/or the collecting of homework papers.

Transition (TR)

Transition refers to the process of going from one phase of the class period to another. Transitions should be coded only if they are noticeable (i.e., involve a minute or more, or are not done smoothly). Transition also involves the period of time from when the class is scheduled to begin and when it actually gets productively under way.

Verbal Problem Solving (VP)

Verbal problem solving refers to the time the teacher spends working on word problems. This phase often includes the teacher demonstrating problem solving strategies, solving problems, having the students estimate answers to verbal problems, and/or having the students solve verbal problems.

Quality Rating Scales for Lesson Component Parts

Junior High Study

The quality of each lesson part coded will be given a quality rating based on a five point scale with 5 = excellent, 4 = very good, 3 = average, 2 = fair, and 1 = poor. The following characterizes some of the key dimensions considered in making these ratings for particular lesson component parts.

Review (RE)

Reviewed concepts and skills previously studied in an interesting, beneficial, and efficient manner.

vs.

Did a very inadequate job of reviewing the concepts and skills associated with previous work.

Mental Computation (MC)

Asked challenging and skill-building mental computation exercises in an interesting manner or format.

vs.

Asked mental computation exercises that were of inappropriate difficulty. The work with mental computation was inefficient, not well organized, and uninteresting.

Homework (HW)

Dealt with the homework in a very efficient, effective, interesting, and beneficial manner, without spending an unnecessarily long amount of time on it.

vs.

Dealt with the homework in a very routine and inefficient manner.

Development (DE)

In development the teacher

1. briefly focuses on prerequisite skills and concepts,
2. focuses on meaning and promotion of student understanding by using lively explanations, demonstrations, process explanations, illustrations, etc.,
3. assesses student comprehension by using process/product questions (i.e., active interaction).
4. repeats and elaborates on the meaning portion as necessary.

The above very accurately describes the teacher's actions

vs.

The above does not describe the teacher's actions or describes them in a very minimal way.

Practice Seatwork (PS)

Provided uninterrupted, successful practice, in which everyone was involved immediately and then sustained involvement. Students were alerted that their work would be checked and they were held accountable for it.

vs.

The seatwork was handled inefficiently.

Practice Boardwork (PB)

Made sure that everyone (whether at the board or at their seat) got involved immediately and maintained involvement.

vs.

The practice boardwork was not effective and was handled inefficiently.

Verbal Problem Solving (VP)

The session was efficiently conducted. Appropriate problems were used, interest was maintained, strategies were discussed, solution methods were discussed and demonstrated, and there was an opportunity for questions.

vs.

The verbal problem solving session was poorly conducted and resulted in very little benefit to the students.

Transition (TR)

The transition was very smooth and efficient. Momentum and interest were maintained and though in transition, time was not wasted.

vs.

The transition was unnecessarily long, boring and unprofitable, such that valuable time was wasted.

Description of High Inference Scales

Junior High Mathematics Study

Clarity

Clarity refers to the degree to which the teacher's presentation of material and his substantive interactions with students are understood by them.

5 Very high clarity. The teacher's explanations are easy to understand and pupil questions are adequately answered. The teacher seems aware of the pupil's levels, sensing problems they are having or may have.

4 High clarity. Between moderate and very high.

3 Moderate clarity. The teacher seems to be understood by most pupils, but not all of the time. Sometimes the teacher is confusing and vague.

2 Low clarity. Between very low and moderate.

1 Very low clarity. Pupils seem very confused by the presentation. The teacher cannot answer the pupils' questions, or answers them in an unclear manner by using concepts and terms the pupils are apparently unfamiliar with or by being overly complex and ambiguous.

Enthusiasm

This scale is used to judge the extent to which the teacher displays interest, vitality, and involvement in his subject and his instruction.

5 Very high enthusiasm. The teacher is stimulating, energetic, and very alert. He seems interested and involved in what he is teaching; moves around, gestures, inflects voice.

4 High enthusiasm. Between moderate and very high.

3 Moderate enthusiasm. Occasionally the teacher seems interested and involved; some display of activity, such as gesturing. Sometimes the teacher is dull, routine, and lacking in vigor.

2 Low enthusiasm. Between very low and moderate.

1 Very low enthusiasm. The teacher's behavior is lethargic, dull, routine; a minimum of vocal inflection, gesturing, movement, or change in facial features. The teacher appears to lack interest in what he is doing.

Managerial

This scale is used to judge the degree of effectiveness of the managerial skills displayed by the teacher.

5 Very high management. The teacher is an effective manager. She structures, maintains and monitors learning activities. She runs the class with a minimum of disruptions.

4 High management. Between moderate and very high.

3 Moderate management. Occasionally the teacher is an effective manager; some display of structuring, maintaining and monitoring learning is present. Sometimes she manages the room very ineffectively and allows too many disruptions.

2 Low management. Between very low and moderate.

1 Very low management. The teacher manages the classroom very inefficiently and ineffectively. Too many disruptions are tolerated.

Accomplishment Index

5 Very high accomplishment. The teacher accomplishes a remarkable amount during the class period in terms of the number of examples used, the number of problems worked, the amount of material covered, and so on, in relation to what seemed possible.

4 High accomplishment. Between moderate and very high.

3 Moderate accomplishment. At times the teacher seems to be accomplishing a lot but at other times things drag with very little being accomplished, in relation to what seemed possible.

2 Low accomplishment. Between very low and moderate.

1 Very low accomplishment. The teacher accomplishes very little compared to what seemed possible.

Interaction Index (excluding seatwork)

5 Very high participation. The teacher's behavior patterns elicit a large amount of active student participation and interaction. The students willingly and enthusiastically take an active part in the lesson. There are a lot of self-initiated questions and teacher questions. Frequent question and answer sessions are observed.

4 High participation. Between moderate and very high.

3 Moderate participation. Occasionally there is some active student participation and interaction, but at other times there is very little.

2 Low participation. Between very low and moderate.

1 Very low participation. There is little or no active student participation and interaction during the lesson.

Implementation.

Use this scale to indicate how well the teacher implemented the prescribed treatment behaviors (circle one). See the attached page for a description of the major parts of the treatment program.

5	4	3	2	1
Implemented all major components of the program	Implemented most of the major components	Implemented about one-half of the program	Implemented some of the program components	Implemented very little of the program

Overall Quality.

Describe the teacher's overall quality based on the entire class period.

5	4	3	2	1
Excellent	Very Good	Average	Fair	Poor

Treatment Program Description

Junior High Study

The treatment involves the following:

1. The beginning portion of the lesson (first 12 minutes) should have three parts:
    a. A brief review,
    b. Checking of homework, and
    c. 3-5 minutes of mental computation exercises.
2. Approximately 10 minutes daily should be spent on verbal problem solving.
3. Approximately 20 minutes daily be devoted to developing conceptual understanding of mathematics.
4. Approximately 30 minutes worth of homework be assigned each night except Friday.
5. Approximately 10-15 minutes of seatwork be given daily to provide uninterrupted successful practice.
6. The first 20 minutes each Monday should be spent on review and the full period be used for review every fourth Monday.
7. The teacher should get everyone involved in the seatwork immediately and keep them involved.
8. The teacher should make the students accountable for the seatwork.

NAME: SCHOOL: WEEKS BEGINNING &

Put a check(✓) in the columns that apply and fill in other appropriate information

DAY OF WEEK	BRIEF REVIEW	HOMEWORK CHECKED	MENTAL COMPUTATION	ABOUT 20 MINUTES ON DEVELOPMENT	ABOUT 10 MINUTES ON VERBAL PROBLEMS	STRATEGY USED					SEATWORK PAGE/PROBLEM #	HOMEWORK PAGE/PROB.#	COMMENTS*
						PROBLEMS WITH-OUT NUMBERS	WRITING VERBAL PROBLEMS	ESTIMATING THE ANSWER	READING VERBAL PROBLEMS	WRITING OPEN SENTENCES			
MONDAY													
TUESDAY													
WEDNESDAY													
THURSDAY													
FRIDAY													
MONDAY													
TUESDAY													
WEDNESDAY													
THURSDAY													
FRIDAY													

* Please note review days,test days,assemblies,shortened periods,quizzes,etc.

DESCRIPTIONS OF TIME ALLOCATIONS

Junior High Mathematics Study

DAILY SCHEDULE FOR TUESDAY - FRIDAY

Phase 1: Introduction should include:

1. 1-2 minutes on review of concepts and skills associated with the homework.
2. 3-5 minutes spent checking, collecting, and dealing with homework assignments.
3. 2-5 minutes spent on mental computation.

Phase 2: Approximately 28 minutes be devoted to development of the mathematics of the lesson. This phase should include four parts:

1. Verbal problem solving (10 minutes).
2. Comprehension phase
3. Teacher questions to assess comprehension
4. Controlled practice

Phase 3: Approximately 15 minutes be spent on seatwork.

Phase 4: Assign Homework (Monday - Thursday).

SCHEDULE FOR MONDAY

Phase 1: The first one-half of each Monday's class period (about 25 minutes) should be devoted to review/maintenance.

Phase 2: Approximately 20 minutes on development.

1. Verbal problem solving (10 minutes).
2. Comprehension phase
3. Teacher questions to assess comprehension
4. Controlled practice

Phase 3: Approximately 10 minutes of seatwork.

Phase 4: Assign Homework

SCHEDULE FOR EVERY FOURTH MONDAY

The entire mathematics period should be devoted to a cumulative review/maintenance session, every fourth Monday.

TIME SCHEDULE

Junior High Mathematics Study

INTRODUCTORY PHASE (12 minutes)

Review of concepts and skills associated with the homework (1-2 minutes).

Checking, collecting, and dealing with homework assignments (3-5 minutes).

Mental computation (3-5 minutes).

DEVELOPMENT (28 minutes)

Verbal Problem Solving (10 minutes).

Comprehension Phase

Teacher questions to assess comprehension.

Controlled Practice.

SEATWORK (15 minutes)

Appendix 10

Achievement Tests Used in Junior High Project

Pre Test

Key

NAME: ______________________________ GRADE: ____

TEACHER: ___________________________ CLASS PERIOD: ____

JUNIOR HIGH SCHOOL: ____

PLEASE DO NOT TURN THE PAGE UNTIL YOU ARE TOLD.

DO NOT BEGIN UNTIL DIRECTED

ADDITION

DO ALL OF YOUR WORK ON THIS SIDE

Express answers in their lowest terms
Example: $\frac{2}{4} = \frac{1}{2}$

1. $56 + 63 =$ 119

2. $\begin{array}{r} 98 \\ +\ 76 \\ \hline 174 \end{array}$

3. $\begin{array}{r} 387 \\ +\ 34 \\ \hline 421 \end{array}$

4. $\frac{2}{5} + \frac{1}{5} =$ 3/5

5. $\frac{1}{2} + \frac{1}{3} =$ 5/6

2pts 1pt.

6. $\frac{9}{16} + \frac{7}{8} =$ 17/16 23/16

7. $.72 + 1.6 =$ 2.32

8. $26 + N = 49$
   $N =$ 23

9. $(4X3) + (6X8) =$ 60

ADDITION Continued

DO ALL OF YOUR WORK ON THIS SIDE

10. 3 feet 6 inches
+ 2 feet 8 inches
6 feet 2 inches

11. $\frac{16}{27} + \frac{2}{9} + \frac{2}{3} = \underline{1\frac{13}{27}}$ 2pts. $\frac{40}{27}$ 1pt.

12. $-4 + 8 + 16 = \underline{20}$

13. $4^2 + 3^3 = \underline{43}$

14. $(3) + (-5) = \underline{-2}$

15. $3\frac{4}{8} + \frac{3}{12} + 7\frac{1}{24} = \underline{10\frac{19}{24}}$ 2pts. $\frac{259}{24}$ 1pt.

16. $(-37) + 48 + (-13) = \underline{-2}$

PRIME FACTORS

Circle the prime factors.

17. [✓ 2], 4, [✓ 5], 10, 12, [✓ 13], [✓ 17] 22

4pts. possible - 1pt. off for each error up to four

Give the lowest common multiple (prime factors) of these numbers.

18. Lowest prime factors of 12 = 2, 3 or 2 or 2,2,3

19. Lowest prime factors of 15 = 3,5 or 3

20. Lowest prime factors of 30 = 2,3,5 or 2

SUBTRACTION

DO ALL OF YOUR WORK ON THIS SIDE

Express answers in their lowest terms.

21. $\begin{array}{r} 214 \\ -\ 87 \\ \hline 127 \end{array}$

22. $\frac{8}{9} - \frac{6}{9} =$ 2/9

23. $\frac{3}{4} - \frac{1}{2} =$ 1/4

24. $\frac{15}{32} - \frac{1}{4} =$ 7/32

25. 1.28 - .39 = .89

26. 82 - N = 48

N = 34

27. (7X9) - (6X3) = 45

28. 4 feet 3 inches

- 3 feet 4 inches

11 inches or 0 feet 11 inches

2pts. 1pt.

29. $1\frac{3}{8} - \frac{5}{8} =$ 3/4 6/8

SUBTRACTION Continued

DO ALL OF YOUR WORK ON THIS SIDE

30. $(-13) - (4) =$ -17

31. $\frac{15}{10} - 1\frac{1}{5} =$ $3/10$

32. $(-28) - (-7) =$ -21

2pts. 1pt.

33. $3\frac{3}{7} - 1\frac{2}{3} =$ $1\frac{16}{21}$ $37/21$

34. $(-10) - (+20) =$ -30

2pts. 1pt.

35. $3\frac{4}{5} - 1\frac{7}{5} =$ $1\,2/5$ $7/5$

MULTIPLICATION

Express answers in their lowest terms.

36. 68×7
476

37. 57×74
4218

38. 698×93
64, 914

MULTIPLICATION Continued

DO ALL OF YOUR WORK ON THIS SIDE

2pts. 1pt.

39. $\frac{3}{4}$ X $\frac{2}{3}$ = 1/2 6/12

2pts. 1pt.

40. $\frac{7}{12}$ X $\frac{3}{5}$ = 7/20 21/60

41. 15% of 400 = 60

42.
$$\begin{array}{r} 600 \\ \underline{\text{X}.17} \end{array}$$
102 or 102.00

43. 4^3 = 64

44. (3X3) X (5X8) = 360

45. (.25 X 172) X 4 = 172

46. $\frac{7}{8}$ = 87.5 % or 87½%

47. $\frac{4}{5}$ = 80 %

48. 2^2 X 4^2 = 64

49. -8 X -4 = 32

DIVISION

DO ALL OF YOUR WORK ON THIS SIDE

Express answers in their lowest terms.

50. $\begin{array}{r} 51 \\ 86\overline{)4386} \end{array}$

51. $\begin{array}{r} 293 \\ 58\overline{)16{,}994} \end{array}$

52. $\begin{array}{r} 130 \\ .17\overline{)22.1} \end{array}$

53. $\frac{3}{4} \div \frac{1}{3} = \overset{\text{2pts.}}{\underline{2\,\frac{1}{4}}}$ $\overset{\text{1pt.}}{9/4}$

54. $\frac{1}{8} \div \frac{5}{6} = \overset{\text{2pts.}}{\underline{3/20}}$ $\overset{\text{1pt.}}{6/40}$

55. 17 = __20__ % of 85

56. 20 = __25__ % of 80

57. 90 = __45__ % of 200

DIVISION Continued

DO ALL OF YOUR WORK ON THIS SIDE

58. (9X8) ÷ (2X4) = 9

2pts. 1pt.

59. $3\frac{2}{3} \div 4\frac{1}{8} =$ 8/9 88/99

60. $\sqrt{49} =$ 7

61. $\sqrt{25} =$ 5

Write these decimals as fractions:

$.10 = \frac{1}{10}$

62. 2.375 = $2\frac{375}{1000}$ or $\frac{2375}{1000}$ or $\frac{19}{8}$ or $2\,3/8$ or $2\frac{15}{40}$ or $2\frac{75}{200}$

63. 0.92 = $\frac{92}{100}$ or $\frac{23}{25}$

64. 1.15 = $\frac{115}{100}$ or $\frac{23}{20}$ or $1\frac{15}{100}$ or $1\frac{3}{20}$

Write these fractions as decimals:

$\frac{1}{10} = .10$

65. $\frac{17}{4} =$ 4.25

DIVISION Continued

DO ALL OF YOUR WORK ON THIS SIDE

66. $\frac{5}{8}$ = .625

67. $\frac{405}{450}$ = .90 or .9

WORD PROBLEMS

68. How much will a dozen apples cost if 3 apples cost 30¢?

$1.20 or 120¢ or 1.20

69. How much can we spend for the class party? The parents gave us $5.00 and 25 children brought a dime each.

$7.50 or 7.50 or 750¢

70. Jim found 25 golf balls. He will keep 10 and give 1/3 of what is left to each of 3 friends. How many will each friend get?

5 or 5 balls

71. Doug bought a bike for $30 and sold it for $40. Then he bought it back for $45 and sold it again for $50. How much profit did he make altogether?

$15 or 15.00 or 15

WORD PROBLEMS Continued

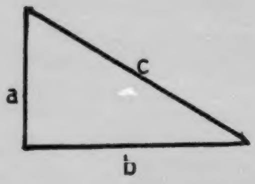
DO ALL OF YOUR WORK ON THIS SIDE

72. A coat was reduced 25%. The price now is $30.00. What was the original price?

$40.00 or $40 or 40

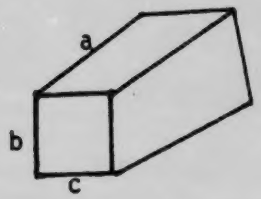
GEOMETRY

73.



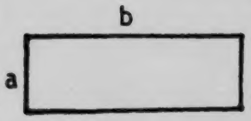
a = 6
b = 8
c = 10

74.



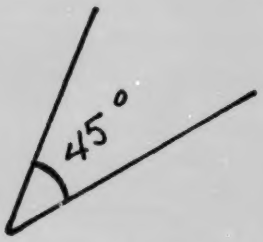
a = 4
b = 3
c = 3
What is the volume?
36

75.



a = 10
b = 12
What is the area?
120

76.

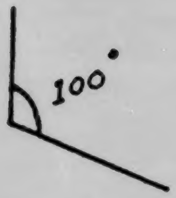


This is: b or acute angle

a) = a right angle.
[✓ b) = an acute angle.]
or
c) = an isosceles angle.
d) = an obtuse angle.

DO ALL OF YOUR WORK ON THIS SIDE

77.



This is: d or obtuse angle

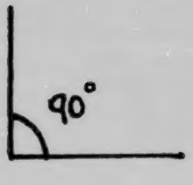
a) = a right angle.

b) = an acute angle.

c) = an isosceles angle.

or [✓ d) = an obtuse angle.]

78.



This is: a or right angle

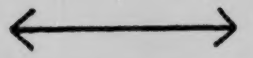
or [✓ a) = a right angle.]

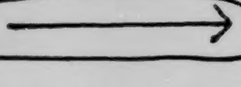
b) = an acute angle.

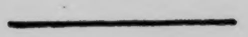
c) = an isosceles angle.

d) = an obtuse angle.

79. A ray is: (Circle the answer.)

a) 

[✓ b)] 

c) 

Write these percents as decimals.

80. 10% = .1 or .10

81. $33\frac{1}{3}\%$ = .333 or .$\overline{3}$ or .$\overline{33}$ or .333... etc.

Write these fractions as decimals.

DO ALL OF YOUR WORK ON THIS SIDE

82. $\frac{2}{3}$ = .66 or .67 or .$\overline{66}$ or .$\bar{6}$ etc.

83. $\frac{3}{4}$ = .75

Write these fractions as decimals and percents.

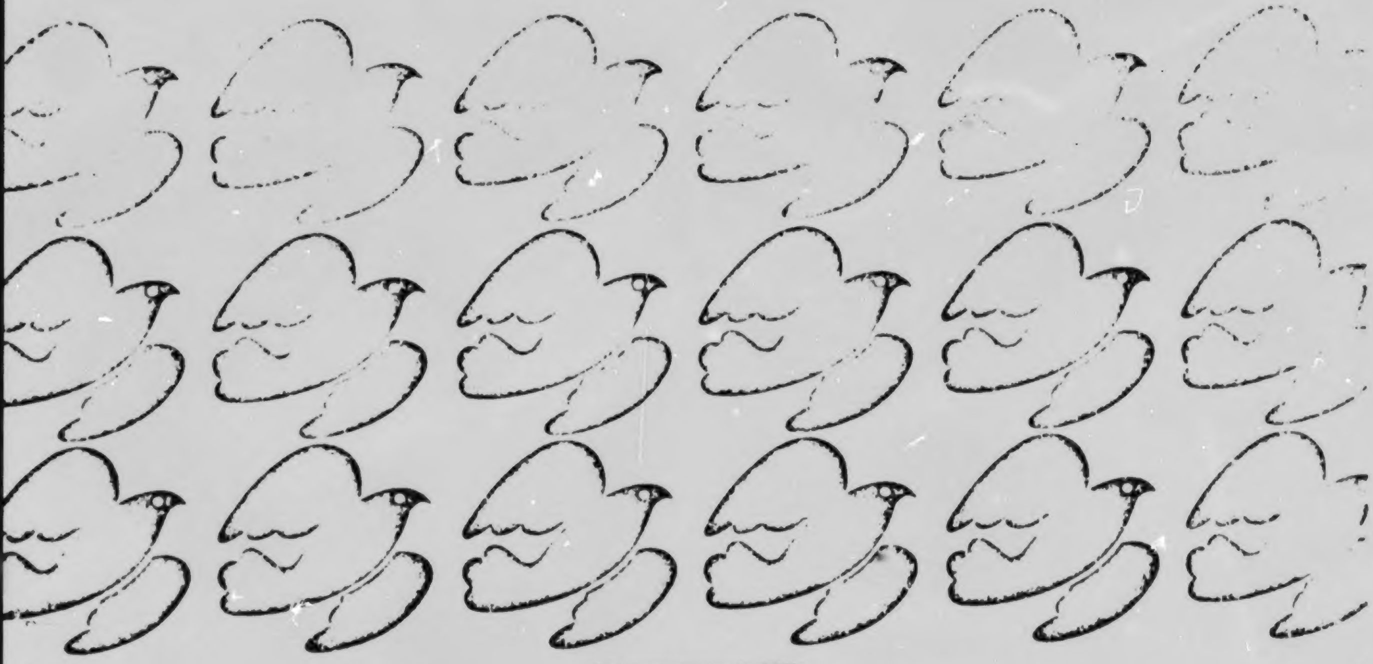
84. $\frac{1}{6}$ = 16 2/3 or 16.7 % = .167 or .1$\bar{6}$ (2pts.)

85. $\frac{1}{4}$ = 25 % = .25 (2pts.)

Post Test

LEVEL F / Form 1

SRA Achievement Series



SRA

SAMPLE **S1.** 1006 is read

A. one hundred six
B. one hundred sixty
C. one thousand six
D. ten thousand six

Directions: Answer these questions.

1. What is the place value of 4 in 14,050,390?

A. Ten thousands
B. Hundred thousands
C. Millions
D. Ten millions

2. 589 + 793 is closest to

A. 600 + 800
B. 600 + 700
C. 500 + 800
D. 500 + 700

3. 30,680 is the same as

A. $(3 \times 1000) + (6 \times 100) + (8 \times 10)$
B. $(3 \times 10{,}000) + (6 \times 100) + (8 \times 10)$
C. $(3 \times 100{,}000) + (6 \times 1000) + (8 \times 10)$
D. $(3 \times 100{,}000) + (6 \times 100) + (8 \times 10)$

4. 87,439 rounded to the nearest hundred is

A. 87,000
B. 87,400
C. 87,440
D. 87,500

5. What number belongs in the box?

$$123 + \square = 232$$

A. 355
B. 255
C. 111
D. 109

6. Which is the next number in the pattern?

4, 5, 7, 10

A. 11
B. 12
C. 14
D. 16

7. The least common multiple of 10 and 15 is

A. 30
B. 60
C. 90
D. 150

8. The greatest common factor of 30 and 75 is

A. 5
B. 6
C. 15
D. 25

9. The prime factorization of 48 is

A. $2 \times 2 \times 2 \times 2 \times 3$
B. $2 \times 2 \times 2 \times 3 \times 3$
C. $2 \times 2 \times 3 \times 4$
D. $2 \times 3 \times 8$

GO ON TO THE NEXT PAGE.

10. $\frac{6}{9}$ is equal to

A. $\frac{3}{4}$
B. $\frac{2}{3}$
C. $\frac{1}{3}$
D. $\frac{1}{2}$

11. Which symbol belongs in the circle?

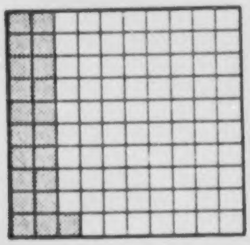
$\frac{2}{9}$ ○ $\frac{1}{4}$

A. <
B. >
C. =
D. ≥

12. $1\frac{4}{5}$ is equal to

A. $\frac{14}{5}$
B. $\frac{9}{5}$
C. $\frac{6}{5}$
D. $\frac{5}{9}$

13. Which decimal tells how much is shaded?



A. 2.1
B. .21
C. .201
D. .021

14. What is the place value of 2 in 5.26?

A. Tenths
B. Hundredths
C. Thousandths
D. Tens

15. What is the place value of 7 in .6279?

A. Ten thousandths
B. Thousandths
C. Hundredths
D. Tenths

16. $2\frac{3}{100}$ is equal to

A. .023
B. .23
C. 2.03
D. 2.3

17. .075 is equal to

A. $\frac{75}{10}$
B. $\frac{75}{100}$
C. $\frac{75}{1000}$
D. $\frac{75}{10,000}$

18. Which symbol belongs in the circle?

.019 ○ .02

A. <
B. >
C. =
D. ≥

GO ON TO THE NEXT PAGE.

19. What is the solution of this proportion?

$$\frac{9}{12} = \frac{6}{x}$$

A. $4\frac{1}{2}$

B. 8

C. 9

D. 18

20. 85% is equal to

A. .085
B. .85
C. 8.5
D. 85

21. What number belongs in the box?

$$^+2 + {}^-2 = \square$$

A. $^-4$

B. $^-1$

C. 0

D. $^+4$

22. Which of these equations has 4 as its solution?

A. $12 - n = 3$

B. $n - 12 = 3$

C. $\frac{n}{3} = 12$

D. $3 \times n = 12$

23. What is the solution?

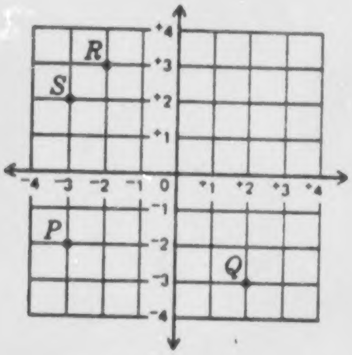
$$x - 5 = 10$$

A. 2
B. 5
C. 15
D. 50

24. 6^2 is equal to

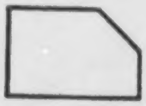
A. 216
B. 36
C. 12
D. 8

25. Which point shown below corresponds to ($^-3$, $^+2$)?

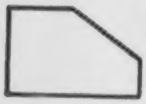


A. Point *P*
B. Point *Q*
C. Point *R*
D. Point *S*

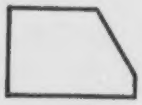
26. Which polygon is congruent to this polygon?



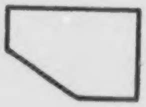
A.



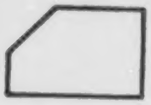
B.



C.



D.



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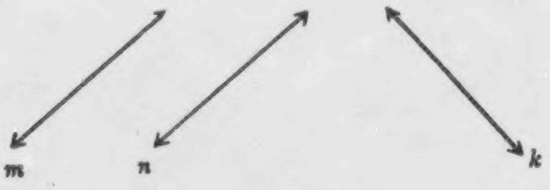
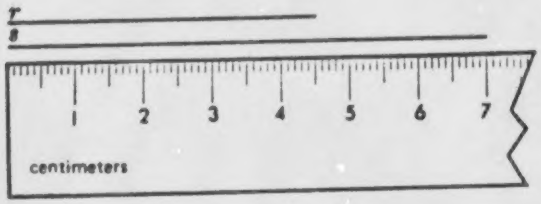
27. Line *k* is perpendicular to line *n*.
Line *m* is parallel to line *n*.



Therefore, it must be true that

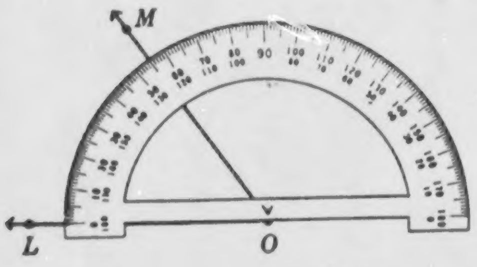
A. line *k* is perpendicular to line *m*
B. line *k* is parallel to line *m*
C. line *k* is parallel to line *n*
D. line *m* is perpendicular to line *n*

28. Line segment *s* is how many centimeters longer than line segment *r*?



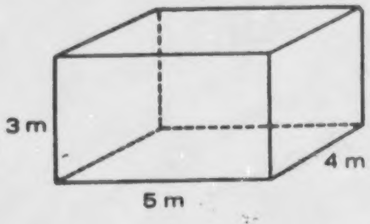
A. 1.5
B. 2
C. 2.5
D. 3

29. What is the degree measure of angle *LOM*?



A. 135°
B. 125°
C. 65°
D. 55°

30. What is the volume of this rectangular prism?



A. 12 cubic meters
B. 20 cubic meters
C. 27 cubic meters
D. 60 cubic meters

STOP HERE.
END OF TEST.

SAMPLE

S1. 48
− 23

A. 15
B. 21
C. 35
D. 71
E. None of these

Directions: Work these problems.

1. 403
3847
+ 629

A. 4979
B. 4969
C. 4879
D. 4869
E. None of these

2. 8392
− 5794

A. 3598
B. 3406
C. 2608
D. 2598
E. None of these

3. 4005
− 918

A. 3087
B. 3097
C. 3187
D. 4913
E. None of these

4. 674
× 3

A. 2022
B. 2012
C. 1922
D. 1812
E. None of these

5. 6184
× 7

A. 43,768
B. 42,788
C. 42,768
D. 42,288
E. None of these

6. 41
× 25

A. 1045
B. 1025
C. 845
D. 287
E. None of these

7. 285 × 60 =

A. 1680
B. 1710
C. 16,800
D. 17,100
E. None of these

8. 384
× 507

A. 21,888
B. 194,188
C. 194,688
D. 194,888
E. None of these

9. $9\overline{)432}$

A. 47
B. 47 R8
C. 48
D. 49
E. None of these

10. $4\overline{)9582}$

A. 2395 R2
B. 2393 R3
C. 2385 R2
D. 2383 R3
E. None of these

GO ON TO THE NEXT PAGE

11. $48\overline{)4471}$
- A. 87 R9
- B. 89 R12
- C. 93 R24
- D. 94 R11
- E. None of these

12. $13\overline{)10{,}920}$
- A. 83 R7
- B. 84
- C. 830 R7
- D. 840
- E. None of these

13. $231\overline{)21{,}947}$
- A. 94 R133
- B. 95 R2
- C. 96 R13
- D. 97
- E. None of these

14. $\begin{array}{r} \$24.10 \\ -\ 13.67 \\ \hline \end{array}$
- A. $11.57
- B. $10.57
- C. $10.53
- D. $10.43
- E. None of these

15. $3 \times \$46.08 =$
- A. $128.04
- B. $128.24
- C. $138.04
- D. $138.24
- E. None of these

16. $60 \times \$4.22 =$
- A. $243.20
- B. $253.20
- C. $2432.00
- D. $2532.00
- E. None of these

17. $\$145.25 \div 7 =$
- A. $20.35
- B. $20.75
- C. $27.50
- D. $27.65
- E. None of these

18. $\begin{array}{r} 9.97 \\ +\ 6.042 \\ \hline \end{array}$
- A. 15.012
- B. 15.1012
- C. 15.912
- D. 16.912
- E. None of these

19. $5.4 + 8.23 =$
- A. 8.77
- B. 13.63
- C. 14.63
- D. 87.7
- E. None of these

20. $2.053 + 14.6 + 9.75 =$
- A. 26.403
- B. 26.503
- C. 30.74
- D. 31.74
- E. None of these

21. $\begin{array}{r} 6.41 \\ -\ 1.5 \\ \hline \end{array}$
- A. 4.91
- B. 5.11
- C. 5.91
- D. 7.91
- E. None of these

22. $15.3 - 6.445 =$
- A. 8.865
- B. 8.945
- C. 8.965
- D. 9.145
- E. None of these

GO ON TO THE NEXT PAGE.

23. $$\begin{array}{r} 3.75 \\ \times\ \ 4 \\ \hline \end{array}$$

A. 1.48
B. 1.5
C. 14.8
D. 15
E. None of these

24. $6.7 \times 1.1 =$

A. 1.34
B. 7.37
C. 13.4
D. 73.7
E. None of these

25. $7\overline{)2.065}$

A. .0295
B. .295
C. 2.95
D. 29.5
E. None of these

26. $.0156 \div 1.2 =$

A. .0013
B. .013
C. .13
D. 1.3
E. None of these

27. Find the number that makes the sentence true.

3 is ☐ % of 4.

A. 25
B. 50
C. 65
D. 70
E. None of these

Directions: Work these problems. Express fractions in lowest terms.

28. $\frac{7}{15} + \frac{4}{15} =$

A. $\frac{2}{3}$
B. $\frac{1}{5}$
C. $\frac{11}{15}$
D. $\frac{11}{30}$
E. None of these

29. $\frac{2}{3} + \frac{2}{5} =$

A. $\frac{17}{15}$
B. $\frac{16}{15}$
C. $\frac{4}{15}$
D. $\frac{1}{2}$
E. None of these

30. $2\frac{3}{5} + 6\frac{3}{10} =$

A. $8\frac{2}{5}$
B. $8\frac{4}{5}$
C. $8\frac{7}{10}$
D. $9\frac{1}{10}$
E. None of these

31. $$\begin{array}{r} 3\frac{3}{4} \\ +\ 9\frac{5}{8} \\ \hline \end{array}$$

A. $12\frac{1}{8}$
B. $12\frac{2}{3}$
C. 13
D. $13\frac{3}{8}$
E. None of these

GO ON TO THE NEXT PAGE.

32. $\frac{3}{4} - \frac{3}{5} =$

A. 0

B. $\frac{1}{5}$

C. $\frac{1}{20}$

D. $\frac{7}{20}$

E. None of these

33. $\frac{8}{9} - \frac{2}{3} =$

A. $\frac{2}{3}$

B. $\frac{2}{9}$

C. $\frac{4}{9}$

D. 1

E. None of these

34. $\frac{5}{6} - \frac{1}{4} =$

A. $\frac{7}{12}$

B. $\frac{5}{12}$

C. $\frac{3}{4}$

D. $\frac{1}{3}$

E. None of these

35.

$$\begin{array}{r} 8\frac{2}{3} \\ -\ 1\frac{4}{9} \\ \hline \end{array}$$

A. $6\frac{4}{9}$

B. $7\frac{1}{9}$

C. $7\frac{2}{9}$

D. $7\frac{4}{9}$

E. None of these

36. $\frac{2}{3} \times 4 =$

A. 3

B. $\frac{8}{3}$

C. $\frac{4}{3}$

D. $\frac{1}{6}$

E. None of these

37. $\frac{2}{5} \times \frac{3}{4} =$

A. $\frac{1}{4}$

B. $\frac{3}{5}$

C. $\frac{3}{10}$

D. $\frac{9}{20}$

E. None of these

38. $3\frac{1}{4} \times 8 =$

A. 28
B. 27
C. 26
D. 25
E. None of these

39. $4 + \frac{3}{4} =$

A. $\frac{17}{3}$

B. $\frac{20}{3}$

C. 3

D. 6

E. None of these

40. $\frac{3}{5} + \frac{9}{10} =$

A. $\frac{2}{3}$

B. $\frac{2}{5}$

C. $\frac{7}{10}$

D. $\frac{27}{50}$

E. None of these

STOP HERE.
END OF TEST.

Mathematics: Problem Solving

SAMPLE

S1. Joan plans to buy some beads to make 4 necklaces. She needs 30 beads for each necklace. How many beads should Joan buy?

A. 34
B. 70
C. 102
D. 120

Directions: Work these problems.

The boys and girls of the Elmwood Community Center had a talent show to raise money for a new roof. Tickets to the show cost $.50. There were 24 girls and 18 boys in the show, and 20 other boys and girls worked on the show but were not in it.

1. How many people were in the talent show?

A. 38
B. 42
C. 44
D. 62

2. All together, 410 tickets to the talent show were sold.

How much money was made from ticket sales?

A. $205
B. $360
C. $460
D. $820

3. Art Hawk sold 12 tickets for the talent show for $.50 each.

How much money did Art collect?

A. $6.00
B. $7.00
C. $11.50
D. $12.50

4. Ana was in a singing group with 5 other people. The group sang 3 songs in the show. They practiced the songs 12 hours a week for 4 weeks.

How many hours did Ana's group practice their songs for the show?

A. 24
B. 36
C. 48
D. 240

5. Greg needed 1.4 meters of blue material and 1.75 meters of white material to make his costume for the show.

To find out how many meters of material Greg needed all together, you should

A. divide
B. multiply
C. subtract
D. add

6. Ramón bought 12 rolls of crepe paper to decorate the gym for the show. The price of the crepe paper was 3 rolls for $.70.

How much did Ramón pay for the crepe paper?

A. $2.10
B. $2.80
C. $6.30
D. $8.40

7. Each of the 12 rolls of crepe paper that Ramón bought contained 10.5 meters of crepe paper.

To find out how many meters of crepe paper there were all together, you should

A. add 12 and 10.5
B. divide 12 by 10.5
C. multiply 10.5 by 12
D. divide 10.5 by 12

8. There were 20 different acts in the talent show. Each act lasted about 6 minutes.

The best estimate of the length of time the 20 acts lasted all together is

A. 1 hour
B. 2 hours
C. 3 hours
D. $3\frac{1}{3}$ hours

GO ON TO THE NEXT PAGE

Nora, Odessa, and David planned a surprise party for Phil on his birthday. Phil and eight other people were expected at the party.

9. David planned to cook 24 hamburgers, each weighing about .25 pound.

   To estimate the number of pounds of hamburger meat he needed, you should find the answer to

   A. 24 + .25
   B. .25 + 24
   C. 24 + .25
   D. 24 × .25

10. Nora, Odessa, and David went shopping. They needed 24 hamburger buns. Hamburger buns came 10 in a package.

    How many packages did they need to buy?

    A. 14
    B. 4
    C. 3
    D. 2

11. Nora planned to make 9 liters of fruit punch for the 12 people coming to the lunch. One liter fills 4 glasses.

    How many glasses of punch did Nora plan to make for each person?

    A. $5\frac{1}{3}$

    B. 3

    C. $2\frac{1}{2}$

    D. 2

12. Nora bought 5 cans of apple juice on sale at 3 cans for $1.23.

    How much did Nora pay for 5 cans?

    A. $.41
    B. $.74
    C. $2.05
    D. $2.24

13. Nora bought fruit juice for $4.18, 3 bottles of ginger ale, several packages of nuts for $1.95, and ice cream for $1.40.

    To find out how much money Nora spent all together, you need to know

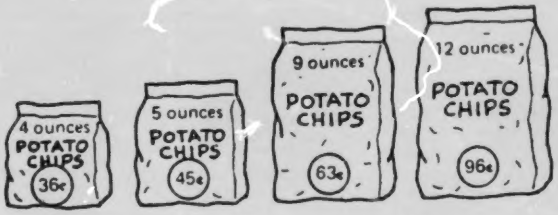
    A. the price of a bottle of ginger ale
    B. how much fruit juice Nora bought
    C. the price of a quart of ice cream
    D. how many packages of nuts Nora bought

14. The fruit punch recipe Nora planned to use called for 50% apple juice, 25% cranberry juice, and 25% ginger ale.

    How much cranberry juice should Nora use to make 9 liters of punch?

    A. 2.25 liters
    B. 2.5 liters
    C. 3.6 liters
    D. 4.25 liters

15. Potato chips came in different-sized bags. The bags looked like this.



    Which bag cost the least per ounce?

    A. The 4-ounce bag
    B. The 5-ounce bag
    C. The 9-ounce bag
    D. The 12-ounce bag

16. Nora, David, and Odessa spent a total of $12.30.

    What was the average amount spent by a person?

    A. $4.10
    B. $9.30
    C. $15.30
    D. $36.90

GO ON TO THE NEXT PAGE.

Tak and a group of his friends had a picnic at the beach.

17. There were 18 people at the picnic, and $\frac{4}{9}$ of them were girls.

    How many girls were at the picnic?

    A. 2
    B. 4
    C. 8
    D. 10

18. The 18 people had 4 jugs of lemonade. Each jug contained 16 cups of lemonade.

    How many cups of lemonade did they have all together?

    A. 72
    B. 64
    C. 54
    D. 38

19. The group brought enough fruit to the picnic for each of 18 people to have at least 3 pieces.

    What is the least number of pieces of fruit the group could have brought to the picnic?

    A. 72
    B. 63
    C. 54
    D. 21

20. The boys and girls had to travel 20.5 kilometers to get to the beach. They rode a bus for 16 kilometers and walked the rest of the way.

    How far did they walk?

    A. 2.5 kilometers
    B. 4.5 kilometers
    C. 18.9 kilometers
    D. 36.5 kilometers

21. The cost of 18 bus fares was $10.80.

    To find the cost of one bus fare, you should

    A. divide 18 by $10.80
    B. add 18 and $10.80
    C. multiply $10.80 by 18
    D. divide $10.80 by 18

Last summer Ms. Foy took some girls and boys from Cole City to visit Oak Lane Farm for a few days. Estella, Charles, Vincent, Francine, and Lewis visited the farm.

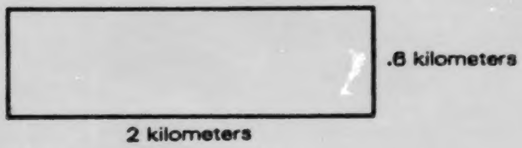
22. Oak Lane Farm is near a village called Fairfield. Estella measured the distance from Cole City to Fairfield on a road map. The distance was 14 centimeters. The scale of the map was

    1 centimeter = 20 kilometers

    How far is Fairfield from Cole City?

    A. 14 kilometers
    B. 28 kilometers
    C. 140 kilometers
    D. 280 kilometers

23. Mr. and Mrs. Marcus own Oak Lane Farm. The farm is shaped like the rectangle shown in the figure below.



    What is the area of Oak Lane Farm?

    A. 1.2 square kilometers
    B. 2.4 square kilometers
    C. 4.6 square kilometers
    D. 5.2 square kilometers

24. Francine and Lewis picked $2\frac{3}{4}$ quarts of blueberries, and Charles picked $1\frac{3}{8}$ quarts of berries.

    To find out how many quarts of berries Francine, Lewis, and Charles picked all together, you should

    A. add
    B. subtract
    C. multiply
    D. divide

GO ON TO THE NEXT PAGE.

25. Estella and Charles tossed a coin to see who would get the next ride on the tractor. Estella chose heads.

What was the probability that the coin would land heads up?

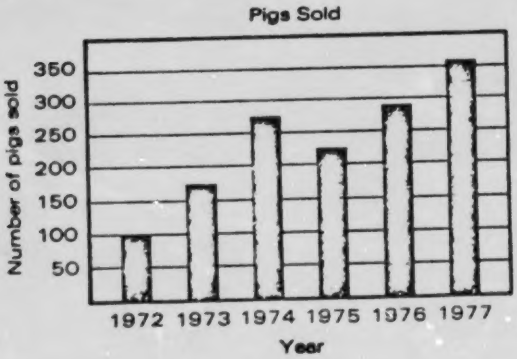
A. 1
B. $\frac{1}{4}$
C. $\frac{1}{3}$
D. $\frac{1}{2}$

26. Mr. Marcus told the visitors that 180 acres of land were planted, and $\frac{2}{3}$ of those 180 acres were planted with corn.

How much land was planted with corn?

A. 60 acres
B. 80 acres
C. 120 acres
D. 270 acres

27. Mr. and Mrs. Marcus raise pigs for the market. The graph below shows how many pigs they sold each year from 1972 to 1977.



The number of pigs sold increased most from

A. 1972 to 1973
B. 1973 to 1974
C. 1975 to 1976
D. 1976 to 1977

28. About how many more pigs were sold in 1975 than in 1972? (See the graph in problem 27.)

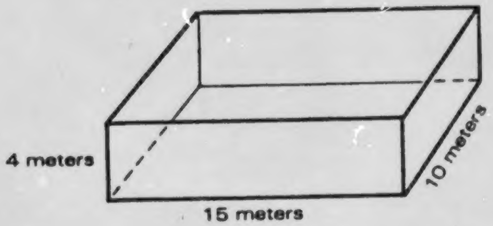
A. 125
B. 150
C. 225
D. 250

29. The area of the vegetable garden at the farm was 330 square meters. Tomato plants took up 10% of the garden space.

How many square meters of land were planted with tomato plants?

A. .33 square meters
B. 3.3 square meters
C. 30 square meters
D. 33 square meters

30. The loft of the barn is used to store hay to feed the animals in winter. The floor of the loft is rectangular, 15 meters long and 10 meters wide. Last autumn the loft was piled 4 meters high with bales of hay. The figure below shows the shape of the space where hay was stored.



What is the volume of the space where the hay was stored?

A. 60 cubic meters
B. 150 cubic meters
C. 300 cubic meters
D. 600 cubic meters

STOP HERE.
END OF TEST.

Appendix 11

Measurement Instruments for Assessing Students' (Aptitude/Attitude Inventory) and Teachers' (Teaching Style Inventory) Beliefs About Mathematics

A. Aptitude/Attitude Inventory

Name ______________________

Boy ________ Girl ______

Teacher's Name ______________

School's Name ______________

ATTITUDE INVENTORY

Directions:

Read each statement and decide if you usually agree or disagree with that statement. If you agree, circle the letter T for True next to the question. If you disagree, circle the letter F for False next to the question.

Please answer every question. Be sure you write your name, your sex, your teacher's name, and your school's name on this sheet. If you have a question, ask your teacher for help.

T F 1. I like to work my math problems with several other students.

T F 2. I always like to choose what math problems to do.

T F 3. I get into trouble in school about once every week.

T F 4. I do not like to work alone.

T F 5. I work harder on math problems that I know will be checked.

T F 6. I need to learn math.

T F 7. I need to be reminded often to get my math assignment done.

T F 8. I want to get good math grades just to show my friends.

T F 9. I sometimes forget to do my assignments.

T F 10. Practicing new math problems with my teacher is a waste of time.

T F 11. I do not need any practice work before I start work on new math problems.

T F 12. I can always remember what I am told to do.

T F 13. I usually finish the easy math problems but not the hard ones.

T F 14. I like my teacher to work a few example problems before I have to do a new problem by myself.

T F 15. I like to learn about math best by listening to my teacher.

T F 16. I will get good math grades this year.

T F 17. I am not good at math games.

T F 18. I usually finish my math assignments.

T F 19. I am good at working math problems in my head.

T F 20. I get into trouble in school about once every week.

T F 21. I like to do math problems in my own way.

T F 22. My teacher really wants me to get good grades in math.

T F 23. I usually do not finish my math assignment.

T F 24. Getting good grades in math is really important to me.

T F 25. I am good at working math problems in my head.

T F 26. I sometimes lose my books and papers.

T F 27. I like to have my parents help me with my math problems.

T F 28. I like to work math problems by myself.

T F 29. I like to learn about math best by reading my book.

T F 30. I always like to choose what math problems to do.

TURN THE PAGE OVER

T. F 31. I like to figure out how to work a new math problem without my teacher's help.

T F 32. I will need math next year.

T F 33. Before I start working new math problems, I like to make sure I can do them.

T F 34. I like to learn about math best by listening to my teacher.

T F 35. I do not like to check my math problems.

T F 36. I like to know if a math assignment will be checked.

T F 37. It is not that important to know math.

T F 38. If I have a question in my math class, I ask the teacher right away.

T F 39. Other subjects are more important than math.

T F 40. My math teacher last year yelled at me a lot.

T F 41. I want to get good grades just for myself.

T F 42. If I find out why I made a mistake on a math problem, I usually do not miss that kind of problem again.

T F 43. I like to be able to choose what our class does in math.

T F 44. I like to have my teacher explain how to work a new math problem.

T F 45. I will get good math grades this year.

T F 46. I do not like to check my math problems.

T F 47. Getting good grades in math is really important to me.

T F 48. If I know my math problems will not be checked, I do not work on them very much.

T F 49. I like to check my math problems to see which problems I missed.

T F 50. I work harder if I know my math problems will be checked.

T F 51. I like to work math problems in my head.

Answer the following questions by circling . . .

1 if you want to answer always
2 if you want to answer most of the time
3 if you want to answer sometimes
4 if you want to answer never

Always	Most of the time	Sometimes	Never	
1	2	3	4	52. Do you like to be in this class?
1	2	3	4	53. Do you have much fun in this class?
1	2	3	4	54. Do most of your close friends like the teacher?
1	2	3	4	55. Does the teacher help you enough?
1	2	3	4	56. Do you learn a lot in this class?
1	2	3	4	57. Do you ever feel like staying away from this class?
1	2	3	4	58. Are you proud to be in this class?
1	2	3	4	59. Do you always do your best in this class?
1	2	3	4	60. Do you talk in class discussions in this class?
1	2	3	4	61. Are most of the students in this class friendly to you?

B. Teaching Style Inventory

Name ______________________________

School ______________________________

Part I CLASSROOM PROCEDURES

Please check the point within each of the following scales which most accurately describes your math class. (If you are teaching math for the first time or your present situation is very different from pervious years, please respond as you anticipate your class will be like this year.) Please respond according to what actually happens, not what you think should happen, or what you would like to have happen. There are no right or wrong answers. Please answer all the questions.

1. Amount of testing

I give a math test about once every three weeks. ____ 1

____ 2

____ 3

____ 4

I give a math test at least once every week. ____ 5

2. Emphasis on enjoyment

Very strong explicit emphasis is put on having a pleasant, happy and friendly time in my math class. ____ 1

____ 2

____ 3

____ 4

Although having an enjoyable time in math is important there is little explicit emphasis on having a pleasant, happy and friendly time in my math class. ____ 5

3. Task emphasis

The importance of getting work done on time and done well is frequently stressed in my class. ____ 1

____ 2

____ 3

____ 4

Students can turn in their work when they are finished. There are no strict deadlines. ____ 5

4. Organization of tasks

Most learning tasks in this class have a step-by-step organization and sequence. ____ 1

____ 2

____ 3

____ 4

Most of the learning tasks in this class are "open-ended" or discovery oriented. ____ 5

5. Commonality

Math learning objectives are the same for all students in the class. ____ 1

____ 2

____ 3

____ 4

Math learning objectives are set for each student separately. ____ 5

6. Problems

Students are encouraged to get a lot of help with their math problems. ____ 1

____ 2

____ 3

____ 4

Students are encouraged to solve their math problems without a lot of teacher help. ____ 5

7. Help with work

Almost all help is initiated by students asking for it. ____ 1

____ 2

____ 3

____ 4

Almost all help is initiated by my seeing the need for it. ____ 5

8. Plan changing

Daily lesson plans are stable, not usually subject to change.

____ 1
____ 2
____ 3
____ 4
____ 5

Daily lesson plans are changed very frequently.

9. Different activities

Many different activities are almost always going on simultaneously during math class.

____ 1
____ 2
____ 3
____ 4
____ 5

Almost all the time the students are all engaged in the same activity during math class.

10. Evaluation standards

The same standards are used for all students.

____ 1
____ 2
____ 3
____ 4
____ 5

Different standards are used for each individual.

11. Evaluation procedures

Evaluation procedures are the same for all students in the class.

____ 1
____ 2
____ 3
____ 4
____ 5

Evaluation procedures are different for each student.

12. Oral presentation

On a typical day, I give an oral presentation for three-fourths of the math time.

____ 1
____ 2
____ 3
____ 4
____ 5

I almost never give an oral math presentation.

13. Peer help

Students frequently help one another during math class.

____ 1
____ 2
____ 3
____ 4
____ 5

Students seldom help one another during math class.

14. Instructional direction

On a typical day, I direct my attention to the math class as a group three-fourths of the time or more.

____ 1
____ 2
____ 3
____ 4
____ 5

On a typical day, I teach or direct my attention to individual students (or small groups) three-fourths of the time or more.

15. Approaches to learning

I encourage students to solve a given math problem the way I have demonstrated.

____ 1
____ 2
____ 3
____ 4
____ 5

I encourage students to solve math problems any way that they desire.

16. Conceptualization

I use conceptual ideas, such as the commutative and associative properties of addition and multiplication to teach math.

____ 1
____ 2
____ 3
____ 4
____ 5

I teach math from a more practical, less theoretical point of view.

17. Inductive-deductive approach

I present a math concepts first then illustrate that concept by working several problems (deductive).

____ 1
____ 2
____ 3
____ 4
____ 5

I present the class with a series of similar problems, then together we develop concepts and methods of solving the problems (inductive).

18. Curriculum organization

The curriculum is organized such that certain topics are repeated (but in more depth) on a regular basis throughout the year.

____ 1
____ 2
____ 3
____ 4
____ 5

Once a certain topic is covered, that same topic is not covered again except during reviews.

19. Transfer

A good deal of time (1/3) is spent trying to teach students to see similarities and differences between new and previously learned math ideas.

____ 1
____ 2
____ 3
____ 4
____ 5

New topics are generally introduced with limited reference to previously learned math ideas.

20. Practicality

Math is taught strictly as a practical subject.

____ 1
____ 2
____ 3
____ 4
____ 5

Math is taught with emphasis on theory.

21. Predictability of student pace

I can usually predict where my students will be in the math textbook in January.

____ 1
____ 2
____ 3
____ 4
____ 5

I can't usually predict where my students will be in the math textbook in January.

22. Student choice

Students have a choice as to what problems or exercises they can do for math practice.

____ 1
____ 2
____ 3
____ 4
____ 5

I decide what problems the students will do for math practice.

23. Pre-assessment

I know a good deal about my students' math abilities before or shortly after the school year starts.

____ 1
____ 2
____ 3
____ 4
____ 5

It usually takes about 9 weeks before I know about my students' math abilities.

24. Motivation

All students are rewarded in the same manner for good work.

____ 1
____ 2
____ 3
____ 4
____ 5

Students are rewarded in different ways for good work.

25. Mobility

Students seldom stay in their seats for the major part of the math lesson.

____ 1
____ 2
____ 3
____ 4
____ 5

Students are generally in the same seat for the math period.

26. Math emphasis

In my math class I emphasize the basic computational skills.

____ 1
____ 2
____ 3
____ 4
____ 5

In my math class I emphasize understanding the concepts underlying mathematics.

27. Study places

Each child works mostly at his own desk during math lesson.

____ 1
____ 2
____ 3
____ 4
____ 5

All math work is divided among a variety of places (centers) in and out of the classroom, with no "home base" seat.

28. Instructional changes

I seldom change my approach throughout the semester (such as lecture-discussion, discovery, etc.).

____ 1
____ 2
____ 3
____ 4
____ 5

I change my approach frequently (from discovery to direct telling or from another method to something different) throughout the semester.

29. Changes

The arrangement of furniture and equipment has changed every week or so, this year.

____ 1
____ 2
____ 3
____ 4
____ 5

The arrangement has changed once or not at all.

30. Rule enforcement

I enforce the classroom rules.

____ 1
____ 2
____ 3
____ 4
____ 5

Students enforce classroom rules.

31. Rule making

I make the classroom rules.

____ 1
____ 2
____ 3
____ 4
____ 5

Students make the classroom rules.

32. Reinforcement

I generally use concrete reinforcers such as stars.

____ 1
____ 2
____ 3
____ 4
____ 5

I generally use verbal praise as reinforcement.

33. Affective objectives

Appreciation of math is of high importance.

____ 1
____ 2
____ 3
____ 4
____ 5

Appreciation of math is not vital.

34. Emphasis on consumer math

Heavy emphasis is placed on consumer math.

____ 1
____ 2
____ 3
____ 4
____ 5

Little emphasis is placed on consumer math.

35. Sex differences

Boys are better in math skills.

___ 1
___ 2
___ 3
___ 4
___ 5

Girls are better in math skills.

36. Divergence from planned lesson

I try hard to stick to the lesson planned for that day during math period.

___ 1
___ 2
___ 3
___ 4
___ 5

If a student raises an interesting question during the math lesson, I may change my whole lesson plan for that day and pursue the student's question.

37. Emphasis on comprehension

Understanding the methodology of why a given method gives the correct answer is important.

___ 1
___ 2
___ 3
___ 4
___ 5

Understanding the methodology is not critical.

38. Exploration

Most of the time is spent drilling the students in math fundamentals.

___ 1
___ 2
___ 3
___ 4
___ 5

Most of the time is spent exploring math-related topics.

39. Pacing

Most math class activities require students to work at about the same pace; topics are expected to be mastered by specific times during the year.

___ 1
___ 2
___ 3
___ 4
___ 5

Each student works at his or her own pace, with no timing restrictions.

Part II TEACHER OPINION

Select the appropriate choice for each statement.

A = Agree

B = Somewhat agree

C = Undecided

D = Somewhat disagree

E = Disagree

40. ___ Teaching math makes me feel secure and at the same time it is stimulating.

41. ___ Teaching multiplication and division is more enjoyable than teaching geometry or fractions.

42. ___ In terms of teaching skill, math, in comparison to other subjects and activities I teach, is a personal strength.

43. ___ Math, in comparison to other subjects and activities I direct, is one of my lesser interests.

44. ___ Math is one of the few areas in which poor readers can do well.

45. ___ My basic function as a math teacher is to convey my knowledge of math to the students in a direct manner.

46. ___ Boys in my class have more interest in math than girls do.

47. ___ Without the assistance of a special teacher (i.e., a specialist in mathematics), the classroom teacher should not be regarded as responsible for the limited progress made by the slowest pupils.

48. ___ Individualization of math instruction seems impractical for actual classroom application.

49. ___ If resources were available, I would prefer total individualization of math instruction rather than group or whole class instruction.

50. ___ I feel I have a good sound background in mathematics.

PART III COMPLETION

51. Most of my students complete ____% or more of all the problems in their textbook associated with each lesson that is taught.

52. As of today, I have _____ students that are discipline problems.

53. When you use practice exercises to reinforce math skills, approximately what percentage are:

____ written work to be done in class

____ written work to be done at home

____ oral work or chalkboard work

____ games or puzzles that illustrate the concept

____ other

100%

54. When some students do poorly on tests or otherwise indicate that they have not understood a unit in math, what are three (3) things you do to improve the situation.

A. ______________________________

B. ______________________________

C. ______________________________

55. On the average I spend about ____ minutes a day developing math concepts and skills and have the children practice these skills through homework and problems ____ minutes a day.

56. This year I teach math ____ days a week for an average of ____ minutes a day.

57. My students should have the opportunity to select and use math materials on a nonstructured basis at least ____ times a week.

58. I assign math work to be done at home about ____ times a week.

59. Sometimes students have difficulty solving story problems. Briefly describe how you help your students solve story problems. (Example: I have pupils make drawings or diagrams to help clarify the problem.)

60. When you correct students' papers, how would you describe the type of marks you most often put on the students' papers? (Example: I mark the problems that are incorrect and provide the correct answer.)

61. How often do you review material already covered? (Example: At the end of the chapter, before vacations, etc.)

62. When I assign students math story problems, I go over the vocabulary in the problem and point out what new words mean about ____% of the time.

63. Before I start presenting the math lesson for the day, I spend about ____ minutes going over the previous lesson.

64. The students in my class make use of or manipulate concrete educational equipment (such as blocks, compasses, rulers, etc.) to aid in understanding math concepts about ____ times a week.

65. I move the students into new material when I feel that all but about ____% of the students are ready.

66. During the year when you start a new math unit that is especially difficult, what do you do differently? (Example: I present the material more slowly than normal and I assure the students they can handle the new material.)

67. Given my present objective and methods of teaching, I feel the ideal class size in math would be ____(number) students and that the maximum number I could teach and still do a good job would be ____(number) students.

68. How many years (including this year) have you taught math to fourth grade students?

_____years

69. How many years (including this year) have you taught in an elementary school setting?

_____years

70. How many hours of college credit in math have you completed (including math methods courses)?

_____hours

71. How many hours of graduate college credit (including courses you may presently be enrolled in) have you completed beyond th[illegible].A. or B.S. degree?

_____hours

72. When math assignments are checked, what percentage would fall into the following categories?

_____ I check the students' papers.
_____ An aide checks the students' papers.
_____ Students check their own work.
_____ Students check each other's work.

_____
100%

73. If you had your choice, what type of ability in math would you prefer to teach? (Check one.)

_____ mostly high ability
_____ mostly average ability
_____ mostly low ability
_____ a mixture of abilities

Appendix 12

Content Analysis: A Comparison of the SRA Achievement Series Level F Form 1 Mathematics Test and Holt School Mathematics

Dr. Jere Confrey

October, 1980

The following strategy was used in analyzing the test and text: First, I classified the test along two dimensions. One dimension was already specified in the test as: concepts, computations and problem, and I accepted their classification at face value. The other dimension was a conventional breakdown of the content into the categories of number sets (whole, fractions, decimals,and ratio and proportion) and other commonly taught topics like geometry and measurement, etc. These categories are roughly ordered from easy to complex.

If a test item involves two content dimensions, I classified it in the high level. Also, under the dimension of concepts, whole numbers could have included place value, estimating patterns and factoring; however, conceptual understanding of whole numbers could have also included basic representations of whole numbers, number line locations or the meaning of operations on whole numbers. None of this was present on the test, and so I separated the categories.

From the categorization, I draw certain conclusions about the fit between the test and the text in terms of relative emphasis. This is done based on the ratio of items devoted to different topics and by comparing this to the order of presentation in the book. The assumption is clear: I am assuming that a teacher will rely on the text, proceeding in order from front to back and is less likely to complete the last sections.

Next, I took the problems in the cells of the taxonomy in order from the upper left corner over and down and checked to see if they are taught in the text, specifying the page. When the text varies from the test, a note is written to describe this misfit.

Finally, I comment on some other factors about the text generally which make it distinctive or which one might want to consider before selecting and using it.

Test	1. Whole numbers	2. Place value	3. Estimation & Rounding	4. Patterns	5. Factoring LCM, GCF	6. Fractions	7. Decimals	8. Percent	9. Ratio & proportion	10. Geometry and coordinate	11. Measurement	12. Probability and Stats	13. Exponents	14. Negative & positive numbers	15. Equations
Concepts		1, 3 14, 15	2, 4	6	7, 8, 9	10,11 12	13,16 17,18	20	19	25, 26 27	28,29, 30		24	21	5,22, 23
Computation	1, 2, 3 4, 5, 6 7, 8, 9 10, 11, 12, 13					28,29, 30,31, 32,33 34,35 36,37 38,39 40	14,15, 16,17, 18,19, 20,21, 22,23, 24,25, 26	27							
Problem Solving	1, 4, 8 10, 16, 19					11,17 24,26	2,3,5, 6,7,9, 13,16, 20,21	14,29	12,15, 22		23,30,	25,27, 28			

Notes on Taxonomy: Relative Emphasis

The test is fairly typical in its emphasis: it tests primarily the topics of whole numbers, fractions and decimals. The other topics of ratio and proportion, measurement and the beginnings of algebra are given cursory attention. This is not the case in the book. Holt School Mathematics begins with whole numbers and then emphasizes decimals, but then spends chapters on geometry, integers and equations, giving, in essence, an introduction to more advanced mathematics courses. It then returns to fractions (rational numbers), percent and real numbers. Finally, the metric system, coordinate geometry and probability and statistics are presented. This focus on preparation for algebra and geometry which is in the text is not reflected on the test which has 5 problems in pre-algebra and 3 on geometry. The book devotes 3 chapters to geometry.

Also, a point which will be emphasized again under individual items, the text makes decimals be the basic number concept. It emphasizes place value in whole numbers and extends this to introduce decimals. Place value for decimals are done through powers of ten. In contrast, the test relies on defining decimals through fractions and tests the conceptual understanding of fractions prior to that of decimals. This difference between test and text is serious in its implications: items on the test which seem obvious if one learns whole numbers, fractions, decimals become far less obvious when fractions are taught in Chapter 9 (p. 212) and decimals in Chapters 2 and 3 (p. 30, 52).

Analysis of Individual Items

Concepts

Classification in Taxonomy	Problem Number Test	Pages in Book	Notes on fit between test item and book instruction
Place Value	1	1	On all these items there is a discrepancy between book and the test. The book never specifically provides exercises on place value, but uses the value of the digit (i.e., 7413: The value of the digit is 400) and writes it in numerals. A chart has it written on it, but it is not used in problems. Even more fundamentally, the book relies heavily on exponents (powers of 10), both positive for whole numbers and negative for decimals. Because of this, for items 1, 14, 15, the student will be required to make an extra step interpreting what 10^{-1} and 10^{-3} mean in words. Students often have difficulty with exponents in algebra ($2^3=6$ is common) and confuse negative numbers and negative exponents, so the choice to go this route in Holt is questionable. The advantage is in changing bases, students are more likely to suggest .13 (base 3 = $1 \times 3^{-1} + 3 \times 3^{-2}$ rather than the error .13(base 3) = 1/3 + 3/30.
	3	1	
	14	37-39	
	15	37-39	
Estimating and Rounding	2	9-10	
	4	2-3	
Patterns	6	not in book	
Factoring LCM, GCF	7	206-207	The only comment I have on this is not any discrepancy in method, but only location in the book. These topics and the following ones on fractions (rational numbers) are placed in Chapt. 9, after integers, and well after decimals. This may reflect an emphasis on decimals, deemphasizing fractions. The test does not reflect this change. Furthermore, the isolation of decimals and fractions leads to an interesting set of questions about the students' understanding of decimals.
	8	204-205	
	9	201-203	

Classification in Taxonomy	Problem Number Test	Pages in Book	Notes on fit between test item and book instruction
Fractions	10	212-3 216-7	The Holt book stresses equivalence here or putting "in simplest terms"
	11	214-215	On comparisons, the book first advises the student to find common denominators and this method will suffice to solve this problem. In addition, the book (top p. 215) shows alternatively a "cross multiplication" method. These methods are not compared and not obviously related to a student. Cross multiplying can be easily confused in trying to decide whether to put number, x der $_2$ first, or visa versa.
	12	218	We are finding, in General Math study, that students memorize the algorithm or "short way" without understanding why it works. Again, the abbreviated neatness of the text obscures the fact that this is not 2 independent methods, but a curtailment of a longer method into a shorter one.
Decimals	13	not taught	As far as I can tell, decimals in this text are taught as strings of numbers with particular values attached (as in money) or off a number line. No representation by a 10x10 "flat" is used)
	16-17	p. 236 #3	As noted under place value, fractions are not stressed as directly connected to decimals. Scientific notation is the dominant mode for expressing decimals in this text. This is a misfit of test to text; how significant it is to students probably depends on whether their previous teaching reinforced this emphasis on metric as independent of fractions.
	18	p. 40-41	Again, avoiding fractions, these students would change .02 to .020 and hence say .019<.020, rather saying either 19/100<20/100 or 19/100< 2/10. In this case, staying totally within the system of decimals, this is relatively straight-forward.
per cent	20	250-251	This comment is on the test. In my experience, students remember to move the decimal pt. two places, but fail to remember whether to go left or right so perhaps 8500 ought to be a foil. (It's even more dramatic when you give them a single digit percent.)

Classification in Taxonomy	Problem Number Test	Pages in Book	Notes on fit between test item and book instruction
Ratio and Proportion	19	p. 232-233	Although the form a/b=c/x is taught in various places (equivalent fractions is one which uses it), it is ratio and proportion where the x is not the product of a simple 'reduction' or 'inflation' of the left side. (However, if one simplifies 9/12 to 3/4, then the problem becomes one like that.) In the book, the term "cross-multiplication" is not used, and means-extremes replaces it. There is no explanation that of the connection of this to cross multiplication used on p. 215 to compare fraction or common denominators.
Geometry and Coordinates	25	p.344-5	This is very late in the text; it is after rational and real numbers, and after geometry. Negative numbers are taught in one dimension,
	26	not in text	The students are taught congruent line segments (p. 82-83) and congruent triangles (296-299). But, no other congruent figures are in the text.
	27	p. 90-103	This section introduces the meaning of perpendicular and parallel as labels and show how to construct them. However, there are no logical exercises in geometry similar to that demanded by the test.
Measurement	28	p.322	There are no problems of this exact form in the metric section or in measurement (p. 64-73). However, I think to do the section on measurement (precision, accuracy and error) successful completion of such a problem would be a prerequisite.
	29	86-88	
	30	p. 314	
Exponents	24	p. 274	
Negative & Positive numbers	21	p.110-115	

Classification in Taxonomy	Problem Number Test	Pages in Book	Notes on fit between test item and book instruction
Equations	5	p.134-148	This book never uses boxes, but introduces the 'variable' and writes of solutions, roots and 'replacements'.
	22	134-148	No questions are asked in the identical form; however, a set of possible 'replacements' is given and student must try each. If students were only asked to solve equations, then this lack of congruence would be a case of reversing. As it is, this would probably cause the student little difficulty.
	23	p.145	This is taught identically
		COMPUTATIONS	
Whole Numbers	1-3	p.6-8	
	4-13	p.12-14	They never do multiple digit x single digit in the text although they test 'x facts'. Also, most examples are displayed vertically on computations, although for estimation, horizontal displays are used.
Fractions	28-29	p.220-1	
	30-31	p.222-3	
	32-35	p.224-5	
	36-38	p.226-7	
	39-40	p.228-9	
Decimals	14-17		The use of money under computation in the text is somewhat surprising since it breaks the trend of computation focusing on number solely and not everyday uses of the numbers. I'd like to know if many students get 14-17 but fail to get equivalent problems in 18-26. Since the book focuses on decimals fundamentally, the 'cushioning' through money seems unnecessary and out-of-place.
	18-26		

Classification in Taxonomy	Problem Number Test	Pages in Book	Notes on fit between test item and book instruction
Percent	27	p.258-9	Again, the use of the box is not in the text. Also, these problems are not written as equations, but are written out. Ex: "8 is what percent of 12?" (p. 259)

PROBLEM SOLVING

Since problem solving involves 1) the type of numbers, 2) the form of context and 3) the strategies on each test problem, although they are grouped by number type, I will identify the other two dimensions and specify if they are addressed in the text. On the test, the implicit definition of problem solving seems to span three dimensions. First, the context must be one which is familiar to students: school plays, picnics, talent shows, etc. are used. Secondly, the problems are grouped. One context is given and a number of questions are asked about that context. Finally, the problems really resemble traditional "word problems" or "verbal problems" in the required strategies. Students are usually expected to select the numbers needed, choose an operation and complete it.

On occasion, they are asked to estimate which always gives exact answers) simply identify the operation and read graphs. Only one time are they asked to identify missing information. Two problems seem to me to be misclassified since they rely on conceptual knowledge of what an average and what odds are.

In the text, the implicit problem solving definition varies. Throughout the text are sections called problem solving, which are either 1) built around themes which is akin to the grouping on the test or 2) applications of a particular skill just taught (also like the text) or 3) career-oriented. However, the text also includes a chapter on problem solving which has exercises on translating between verbal expressions and symbols, writing equations from verbal statement, writing "mini-problems" making up problems from equations, drawing diagrams, reasoning, flow charts, specifying missing information and estimating answers. Only the last two are tested on the SRA series and they each have one problem each.

Two conclusions can be drawn:

1. The test has within it a narrower definition of problem solving than the text, and hence fails to test much of the book's definition of problem solving in chapter seven.

2. However, the book restricts this use of problem solving substantially to one chapter, and for the rest of the book there is a reasonable congruence between text and test. Why the book's authors chose to use such a restrictive definition throughout the rest of the text is a question which needs to be explored.

Topic	Test Problem Number	Page # in Text	Context/Strategy
Whole Numbers	1	The whole number problems are spread throughout the text. Ex. Sec pgs. 8, 17, 27, 48.	Context: School Play Strategy: select appropriate numbers and operations and complete
	4		Context: songs Strategy: select appropriate numbers and operations and complete
	8		Context: Talent show Strategy: estimate and select operation and complete
	10		Context: Picnic Strategy: Select appropriate operation and complete
	18		Context: picnic Strategy: Select appropriate numbers and operations and complete

Topic	Test Problem Number	Page # in Text	Context/Strategy
Fractions	11	Examples: p. 223, 225, 227	Context: Picnic Strategy: Select Appropriate operations and order complete
	17		Context: Picnic Strategy: Select appropriate operation - complete
	24		Context: berry picking Strategy: identify operation
Decimals	2-3	Examples: p. 55, 57 59,61,208	Context: School play Strategy: Select numbers, operations and complete
	5		Context: School play Strategy: Identify operation
	6		Context: School dance Strategy: Select appropriate numbers and operations and complete
	7		Context: School dance Strategy: Identify Operation
	9		Context: Picnic Strategy: Identify Operation
	13		Context: Picnic Strategy: Identify necessary information
	16		Context: Money Strategy: Knowing what an average is.
	20		Context: Beach Strategy: Select operation and complete
	21		Context: Beach Strategy: Identify operation
Per cent	14	Chapter 10 Examples: p.253 257, 263, 267	Context: Picnic Strategy: Select numbers operations and complete

Topic	Test Problem Number	Page # in Text	Context/Strategy
Percent contd.	21		Context: Garden Strategy: Select numbers operations and complete
Ratio and Proportion	12	p. 234	Context: Picnic Strategy: Select operations and order and complete
	15		Context: Picnic Strategy: Select appropriate numbers off a diagram, operations and complete
	22		Context: Maps Strategy: Select appropriate numbers, operations and complete
Measurement	23	p. 74	Context: Farm Strategy: Select appropriate numbers off a diagram, operations and complete
	30	p. 315	Context: Farm Strategy: Select appropriate numbers with a diagram, operations and complete
Probability & Statistics **	25	368-70	Context: Coin tossing Strategy: Know probability concept of coins 50/50
	27	p. 380	Context: Farm Strategy: Read information off a graph and interpret
	28	p. 380	Context: Farm Strategy: Reading information off a graph

**These are all taught at the very end of the text. Often classes will never reach this point during the year.

These final comments are about the text only. Certain strengths and weaknesses of it merit discussion.

1. In the text, the reading level is kept down by minimizing the verbal instructions. By doing so, the authors have left certain connections between 'short' and 'long' methods implicit so that students may well see them as disjoint. This is repeatedly the case in the displays of instruction.
2. The development sections are a good idea to build up the concepts, step by step. In a sense, they reveal by contrast how complicated the displays really are, although they are designed as appearing straightforward.
3. The text sequencing needs consideration. Area is taught algorithmically in Chapter 3, and geometrically with volume and surface area in Chapter 12. By emphasizing decimals, fractions are left until Chapter Nine. The metric system, which I supposed motivated the emphasis on decimals is way back in Chapter 13, graphs are left til Chapter 15, and made secondary to preempting certain topics from prealgebra and geometry, although graphs are likely never to be taught again.
4. The book obviously attempts to omit sex and racial stereotyping. In the career sections, men and women are shown at many careers, and the traditional roles are reversed (i.e., women plumbers, male food processors). In the selection of careers, there is some professional bias--one each of plumbing, exterminators and mechanics, and the rest are programmers, engineers, lawyers, wholesale buyer, economist, food processing technicians. Below, you can see that sexism is controlled reasonably well, but racism still exists. In the problems, names are selected well, and obvious attempts have been

made to alternate sexes in the activities. Page 172 shows two contexts for the equation x + 36 = 100. In one, the girl plays golf, the boy basketball. Both do sports which is an improvement, but in the girl's sport, a skirt is worn and it is fairly passive in contrast to basketball. In my opinion, this is still subtle sex-stereotyping.

Career	Race/Sex of Picture
Employment Counselor	Black female
Plumber	Two white females
Mechanic	White male
Biomedical Engineer	White male
Exterminator	White male
Programmer	White female
Stationary Engineer	White female
Food Processing Technician	Two white males
Lawyer	White male and female
Architects	Oriental male, white male, 2 white females
Hospital Administrator	White female (in nurse's dress)
Teletypist	American Indian (female)
Economists	Group - mixed sex and race
Wholesales	White male and female
Astronomer	White male

Appendix 13 *

Effects of Program and Student Type on Student Time-On-Task Behavior in Eight-Grade Mathematics Classes

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* For more details, the reader can consult Harre (1980)

Dr. Ruthanne Harre, an observer in our junior high study, completed her dissertation (with some small financial assistance from the project) examining the time-on-task behavior of high-, middle-, and low-achieving students in eighth-grade mathematics treatment and control classrooms. She conducted her study using a subset of classrooms that were participating in the general eighth-grade mathematics study. Her data suggest that the involvement rates of students in treatment classrooms were higher than those in control classrooms and thus suggests that one reason why the Missouri Mathematics Program may work is through increased student attention. The following account is taken from the abstract of Dr. Harre's dissertation and three tables are also presented from her dissertation.

Purpose

The purpose of this study was to investigate how different types of students vary in their on-task behavior patterns. Information was also sought which dealt with the interactions among student types and instructional programs, and also with the interactions among student types and phases of the lesson. The correlation between the on-task behavior of six students and that of the whole class was also investigated.

Procedure

The study was conducted during the fall semester of the 1979-80 school year. Twelve eighth grade mathematics teachers from a large mid-western school system volunteered to take part in the study. Of the eight schools represented by these teachers, four were assigned to the treatment conditions and four to the control.

Each classroom was observed on 4 to 7 occasions, during which time data was recorded on how well the treatment was being implemented by the teacher as well as individual student on-task behavior.

Cluster analysis was applied to data obtained from an attitude instrument, as well as sex and achievement data. Four clusters (developed typologies) were obtained: (a) low achievers, (b) high achievers, (c) independent/motivated students, and (d) dependent/unmotivated students, which were employed in the student-treatment analyses.

Each teacher was asked to divide each of her classes approximately into thirds based on achievement. The top third were labeled high achievers, the middle third average achievers, and the remaining third as low achievers. These typologies were used in the student-treatment analyses and also in the student-lesson phase analyses.

For the analyses comparing the on-task behavior of six students with that of the whole class, the three achievement-based typologies were again used. Six students (2 high achievers, 2 average, and 2 low achievers) were randomly selected from within each class.

The treatment teachers were asked to follow the guidelines set forth by the Missouri Mathematics Effective Project (MMEP). This basically involved asking the teachers to pursue a direct instructional model involving active teaching. The class period as detailed by the MMEP breaks down neatly into six lesson phases: (a) mental computation, (b) review, (c) dealing with homework, (d) verbal problem solving, (e) development, and (f) practice seatwork.

Findings

The two-way analysis of variance used to investigate the student type-treatment type interactions found significant main effects on student type ($p = .0007$) and on treatment type ($p = .0001$) when the developed typologies served as the student types. However, the interactive effects were not signficant. Students displayed a significantly higher percentage of on-task behavior during the MMEP treatment conditions than during the control conditions. Dependent students showed a significantly higher rate of on-task behavior than low achievers.

When the achievement-based typologies were used significant main effects were again found for student type ($p = .003$) and for treatment type ($p = .0001$). The interactive effect was significant at the $p = .05$ level. Average achievers displayed a significantly higher rate of on-task behavior than low achievers.

A two-way analysis of variance, with repeated measures, was employed in the student-lesson phase analyses. Here again significant main effects were found on student type ($p = .05$) and on phase ($p = .01$), with the interactive effects not being significant. The phases of the lesson which pertained to mental computation and dealing with homework resulted in higher rates of on-task behavior than did the other four phases.

A high positive correlation ($r = .86$) was found between the on-task behavior of six students and that of the whole class.

Conclusions

The on-task behavior of different types of students was found to vary across treatment instructional programs and also across phases of the lesson.

The study supported the results of two previous studies, which had shown that the MMEP enhanced student achievement, by showing that students were also on-task more during the MMEP treatment as compared to the control conditions.

The findings of this study imply that there is a strong correlation between the on-task behavior of six students and that of the whole class.

Table 1

Analysis of Variance

for Student-Treatment Analyses (Developed Typologies)

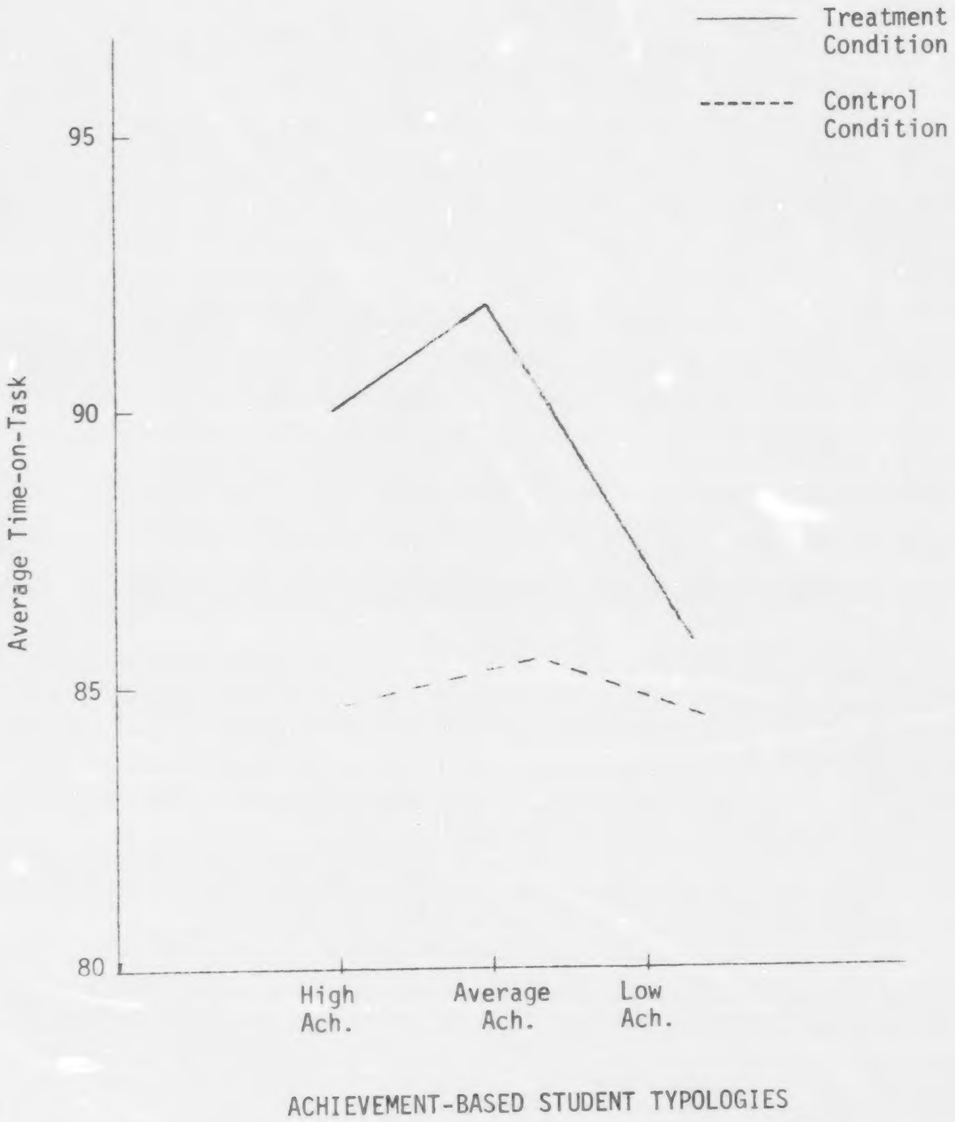
Source	df	MS	F	p
Treatment	1	3773.78	33.74	0.0001
Student	3	660.67	5.91	0.0007
Treatment x Student	3	197.16	1.76	0.1513
Error	608	111.86		

Table 2
Analysis of Variance
for Student-Treatment Analyses
(Achievement-Based Typologies)

Source	df	MS	F	p
Treatment	1	3259.57	30.37	0.0001
Student	2	634.80	5.91	0.0029
Treatment x Student	2	331.44	3.09	0.0463
Error	602	107.33		

Table 3

Student-Treatment Interaction



Appendix 14 *

Verbal Problem Solving Treatment Without the Structure of the Missouri Mathematics Effectiveness Project

* For more details, the reader can consult Engelhardt (1980)

Dr. John Engelhardt, an observer in our junior high study, completed his dissertation research (with some small financial assistance from the project) examining the impact of the verbal problem-solving treatment upon students' performance in sixth-grade classrooms. Earlier (in our Field Experiment II) it had been shown that sixth-grade students' verbal problem-solving abilities were enhanced by exposing their teachers to the general Missouri Mathematics Program and to the verbal problem-solving treatment. The present study tested the verbal problem-solving treatment without the presence of the general treatment program. Dr. Engelhardt's data question the value of the verbal problem treatment in the absence of the more general program. His "feedback" letter to project participants follows and the interested reader can obtain detailed results elsewhere (Engelhardt, 1980).

The following is a summary of the verbal problem-solving research conducted in the public schools from October 23, 1979 through February 6, 1980.

The study was undertaken to experimentally test the effectiveness of a program of systematic instruction in verbal problem solving on the achievement of sixth-grade students. The systematic instruction encompassed a daily time component of 10 minutes (except on days when verbal problem solving was the main focus of the lesson) and five instructional strategies to be used in teaching problem solving--using problems without numbers, writing verbal problems, estimating answers, reading verbal problems, and writing open sentences.

The investigation was designed to answer the following questions: Would the treatment increase problem-solving achievement?, Would the observation influence problem-achievement?, Would the treatment differentially affect the achievement of various groups within the class?, Would the degree of treatment implementation correlate well with residual achievement scores?, and Would student attitude be affected?

Half of the teachers were observed in order to see how well treatment teachers implemented the teaching requests and to measure the extent to which control teachers dealt with problem solving.

After initial instruction during an orientation workshop, the teachers in the treatment group were responsible for maintaining the problem-solving program in their classrooms. The teachers' main references were the Verbal Problem Solving Manual and a Procedure Summary. Beyond these, teachers were to generate the resources necessary for the instructional program. At the conclusion of the experiment all teachers administered a problem-solving test and an attitude scale.

Since no pretests were administered, district data on file from Spring, 1979, testing were used as covariates in the analyses. Those students for whom complete data were on file were considered in the statistical analyses.

Results and Conclusions

With respect to the questions under investigation the following results were noted:

1. The treatment did not make a difference in problem-solving performance either on routine or nonroutine problems. The adjusted mean for the control group was higher than that of the treatment group for routine problems, and the reverse was true for nonroutine problems. Neither difference was significant statistically.

2. Observation was not a factor in problem-solving achievement as the observed group did not differ appreciably from the unobserved group in achievement.

3. The treatment did not have differential effects among high, average, or low groups (within classes) when prior achievement was taken into consideration. However, the average group scored higher than the high group after adjustment.

4. On the attitude toward mathematics scale the control group scored significantly higher than the treatment group, although both groups' scores exhibited a moderately positive attitude toward mathematics.

Based on self-report information from teacher logs, the treatment teachers followed the teaching requests outlined for them at the orientation session prior to the study. The mean number of daily minutes spent on problem solving was 17 and problem solving was covered on 73% of the days school was in session. The averaged to over 10 minutes per school day on problem solving. Treatment teachers reacted favorably to the project although some had reservations as to its potential for widespread acceptance due to increased preparation time and development of materials.

Discussion

Due to the small sample size of 16 teachers, the results were not expected to reach statistical significance. However, the fact that the control group surpassed the treatment group even after adjustment for initial differences was most unexpected. Several plausible considerations suggest themselves.

Previous research supports the use of instructional strategies as a means to increase problem-solving achievement. The present study was designed to make use of regular classroom teachers in a natural school setting, with teachers using available resources. Since teachers were responsible for generating their own resources, some additional preparation was required. For some this may have become a burden. It may be unreasonable to expect teachers to carry out this type of program without additional feedback or material.

Another consideration which clouded interpretation of the results was that the treatment and control groups were not identical on the pretest measures. Control classes exceeded treatment classes by 2/3 of a standard deviation in prior problem-solving achievement and by one standard deviation on knowledge of mathematics concepts. Although adjustment for this difference was made in the statistical work by using analysis of covariance, this does not imply that the groups were comparable. The fact that teachers were randomly assigned to experimental conditions guaranteed a bias-free assignment, not equivalent groups. The treatment group, being lower than the control, may have had a more difficult time increasing problem-solving achievement than would a group equivalent to the control.

A final rationale for the lack of anticipated results comes from a comment made by a teacher during an interview. He mentioned that he thought the program was beneficial but didn't think it would show up on a test due to the short length of the treatment. He indicated that to bring about desirable results would require, in his opinion, a full year of exposure to the program.

Another surprising result was the significant difference on attitude toward mathematics, with the control group scoring higher than the treatment group. This result is based solely on posttesting as no preexperimental attitude measure was administered. Both groups scored between 3 and 4 on a 5-point scale indicating a moderately positive attitude toward mathematics. Though the difference was statistically significant, the educational significance is negligible, given that the group means differed by less than six points out of a possible 130.

It is worth mentioning that in their interviews teachers indicated a favorable reaction to the program. Some noticed changes in their students' reaction to word problems. Several teachers said they thought student attitude toward verbal problems improved and students were not as apprehensive about working verbal problems as they had been.

Implications

Systematic instruction in verbal problem solving is not a sufficient condition to increase problem-solving achievement in a classroom setting when regular teachers use normally available classroom materials. Additional training sessions with teachers may be appropriate and observational feedback may be helpful in keeping teachers on task. A longer treatment period (a full school year) would be desirable. Finally, in order to carry out this type of program perhaps more materials like problem sets should be made available, as this would ease the preparation burden.